Exercise 9.1

Determine order and degree (if defined) of differential equations given in Exercise 1 to 10:

1.

+ sin (y‴)
= 0

Sol. The given D.E. is

+ sin y‴ = 0

The highest order derivative present in the differential equation is order is 4.

and its

The given differential equation is not a polynomial equation in derivatives ($\because$ The term sin y‴ is a T-function of derivative y‴). Therefore degree of this D.E. is not defined.

Ans. Order 4 and degree not defined.

2. y′ + 5y = 0

Sol. The given D.E. is y′ + 5y = 0.

The highest order derivative present in the D.E. is y′

= and

so its order is one. The given D.E. is a polynomial equation in derivatives (y′ here) and the highest power raised to highest order derivative y′ is one, so its degree is one.

Ans. Order 1 and degree 1.

+ 3s

3.

= 0 + 3s

Sol. The given D.E.
is = 0.

The highest order derivative present in the D.E. is and its

order is 2. The given D.E is a polynomial equation in derivatives and the highest power raised to highest order derivative

= 0.

one. Therefore degree of D.E. is 1. Ans. Order 2 and degree 1.

Sol. The given

= 0

+ cos

+ cos
4.
D.E. is

is

The highest order derivative present in the differential equation

is

and its order is 2.

The given D.E. is not a polynomial equation in derivatives ($\because$

The term cos

is a T-function of derivative

Therefore degree of this D.E. is not defined. Ans. Order 2 and degree not defined.

= cos 3x + sin 3x

5.

is

= cos 3x + sin 3x.

).

Sol. The given D.E.

The highest order derivative

present in the D.E. is order is 2.

and its

The given D.E. is a polynomial equation in derivatives and the

is one,

so

highest power raised to
highest order and degree 1.

its degree is 1.
Ans. Order 2 =

Remark. It may be remarked that the terms cos 3x and sin 3x present in the given D.E. are trigonometrical functions (but not T-functions of derivatives).

It may be noted that

is not a polynomial function of

derivatives.

6. $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

Sol. The given D.E. is $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$. ...(i) The highest order derivative present in the D.E. is y''' and its order is 3.

The given D.E. is a polynomial equation in derivatives y''', y'' and y' and the highest power raised to highest order derivative y''' is two, so its degree is 2.

Ans. Order 3 and degree 2.

7. $y''' + 2y'' + y' = 0$

Sol. The given D.E. is $y''' + 2y'' + y' = 0$. ...(i) The highest order derivative present in the D.E. is y‴ and its order is 3.

The given D.E. is a polynomial equation in derivatives y‴, y″ and y′ and the highest power raised to highest order derivative y‴ is one, so its degree is 1.

Ans. Order 3 and degree 1.

8. $y' + y = e^x$

Sol. The given D.E. is $y' + y = e^x$. ...(i) The highest order derivative present in the D.E. is y′ and its order is 1.

The given D.E. is a polynomial equation in derivative y′. (It may be noted that e^x is an exponential function and not a polynomial function but is not an exponential function of derivatives) and the highest power raised to highest order derivative y′ is one, so its degree is 1.

Ans. Order 1 and degree 1.

9. $y'' + (y')^2 + 2y = 0$

Sol. The given D.E. is $y'' + (y')^2 + 2y = 0$. ...(i) The highest order derivative present in the D.E. is y″ and its order is 2.

The given D.E. is a polynomial equation in derivatives y″ and y′ and the highest power raised to highest order derivative y″ is one, so its degree is 1.

Ans. Order 2 and degree 1.

10. $y'' + 2y' + \sin y = 0$

Sol. The given D.E. is $y'' + 2y' + \sin y = 0$. ...(i) The highest order derivative present in the D.E. is y″ and its order is 2.

The given D.E. is a polynomial equation in derivatives y″ and y′. (It may be noted that sin y is not a polynomial function of y, it is a T-function of y but is not a T-function of derivatives) and the highest power raised to highest order derivative y″ is one, so its degree is one.

Ans. Order 2 and degree 1.

11. The degree of the differential equation

$+ \quad + \sin \quad + 1 = 0$ is

(A) 3 (B) 2 (C) 1 (D) Not defined.

Sol. The given D.E. is

$+$

$+ \sin$ $+ 1 = 0$ ...(i)

This D.E. (i) is not a polynomial equation in derivatives.

$\therefore$ Degree of D.E. (i) is not defined.

Answer. Option (D) is the correct answer.

12. The order of the differential equation

$2x^2$ -3 $+ y = 0$ is

(A) 2 (B) 1 (C) 0 (D) Not defined

Sol. The given D.E. is $2x^2$ -3 $+ y = 0$

The highest order derivative present in the differential equation is and its order is 2.

Answer. Order of the given D.E. is 2.

Exercise 9.2

In each of the Exercises 1 to 6 verify that the given functions (explicit) is a

^{x} + 1 : y″ – y′ = 0

solution of the corresponding differential equation: 1. y = e ^{x} + 1 ...(i) To prove: y given, by (i) is a solution of the D.E. y″

Sol. Given: y = e – y′ = 0 ...(ii) From (i), y′ = e ^{x}

^{x} + 0 = e

^{x} and y″ = e^{x} – e

∴ L.H.S. of D.E. (ii) = y″ – y′ = e by ^{x} = 0 = R.H.S. of D.E. (ii) ∴ y given (i) is a solution of D.E. (ii). 2. y = x^2 + 2x + C : y′ – 2x – 2 = 0

Sol. Given: y = x^2 + 2x + C ...(i) To prove: y given by (i) is a solution of the D.E.

y′ – 2x – 2 = 0 ...(ii) From (i), y′ = 2x + 2

∴ L.H.S. of D.E. (ii) = y′ – 2x – 2

= (2x + 2) – 2x – 2 = 2x + 2 – 2x – 2 = 0 = R.H.S. of D.E. (ii) ∴ y given by (i) is a solution of D.E. (ii).

3. y = cos x + C : y′ + sin x = 0

Sol. Given: y = cos x + C ...(i) To prove: y given by (i) is a solution of D.E. y′ + sin x = 0 ...(ii) From (i), y′ = – sin x

∴ L.H.S. of D.E. (ii) = y′ + sin x = – sin x + sin x

= 0 = R.H.S. of D.E. (ii)

∴ y given by (i) is a solution of D.E. (ii).

4. y =

: y′ =

Sol. Given: y = + ...(i) To prove: y given by (i) is a solution of D.E. y′ =

+ ...(ii)

From (i), y′ = + = $(1 + x^2)^{1/2}$

$= (1 + x^2)^{-1/2}$ $(1 + x_2)$ $= (1 + x^2)^{-1/2}$. 2x = + ...(iii) R.H.S. of D.E. (ii) =

+ = + + (By (i))

  = =

=

∴

[redacted]

= y′ [By (iii)] = L.H.S. of D.E. (ii)

∴ y given by (i) is a solution of D.E. (ii).

5. y = Ax : xy′ = y (x ≠ 0)

Sol. Given: y = Ax ...(i) To prove: y given by (i) is a solution of the D.E. xy′ = y (x ≠ 0) ...(ii)

From (i), y′ = A(1) = A

L.H.S. of D.E. (ii) = xy′ = xA

= Ax = y [By (i)] = R.H.S. of D.E. (ii)

∴ y given by (i) is a solution of D.E. (ii).

6. y = x sin x : xy′ = y + x (x ≠ 0 and x > y or x < – y) Sol. Given: y = x

sin x ...(i) To prove: y given by (i) is a solution of D.E.

xy′ = y + x – ...(ii) (x ≠ 0 and x > y or x < – y) From (i), (= y′)

= x (sin x) + sin x x = x cos x + sin x L.H.S. of

D.E. (ii) = xy′ = x (x cos x + sin x)

R.H.S. of D.E. (ii) = y + x [redacted] Putting

$y = x \sin x$ from (i),

$= x^2 \cos x + x \sin x$ ...(iii)

$= x \sin x + x$ [redacted] $= x \sin x + x$ [redacted] $= x \sin x + x$

$= x \sin x + x \,.\, x \cos x$

$= x \sin x + x^2 \cos x = x^2 \cos x + x \sin x$ ...(iv) From (iii) and (iv), L.H.S. of D.E. (ii) = R.H.S. of D.E. (ii) $\therefore$ y given by (i) is a solution of D.E. (ii).

In each of the Exercises 7 to 10, verify that the given functions (Explicit or Implicit) is a solution of the corresponding differential equation:

7. $xy = \log y + C : y' =$ $(xy \neq 1)$

Sol. Given: $xy = \log y + C$ ...(i) To prove that Implicit function given by (i) is a solution of the

D.E. $y' =$

$-$ ...(ii)

Differentiating both sides of (i) w.r.t. x, we have xy'

$y' + 0$

$+ y(1) =$

$\Rightarrow xy' - {}' = -y \Rightarrow y'$ $- = -y$

$\Rightarrow y'$

$- = -y \Rightarrow y'(xy - 1) = -y_2$

$- -$

$\Rightarrow y' = - = - - -$

which is same as differential equation (ii), i.e., Eqn. (ii) is proved. $\therefore$ Function (Implicit) given by (i) is a solution of D.E. (ii). 8. $y - \cos y = x$: $(y \sin y + \cos y + x)\, y' = y$

Sol. Given: $y - \cos y = x$ ...(i) To prove that function given by (i) is a solution of D.E. $(y \sin y + \cos y + x)\, y' = y$ ...(ii) Differentiating both sides of (i) w.r.t. x, we have

$y' + (\sin y)\, y' = 1 \Rightarrow y'(1 + \sin y) = 1$

$\Rightarrow y' =$

$+$ ...(iii)

Putting the value of x from (i) and value of y' from (iii) in L.H.S. of (ii), we have

L.H.S. = $(y \sin y + \cos y + x) y'$

$= (y \sin y + \cos y + y - \cos y)$

$+ = (y \sin y + y)$

$+$

$= y (\sin y + 1)$

$+ = y$ = R.H.S. of (ii).

$\therefore$ The function given by (i) is a solution of D.E. (ii). 9. $x + y = \tan^{-1} y : y^2 y' + y^2 + 1 = 0$

Sol. Given: $x + y = \tan^{-1} y$ ...(i) To prove that function given by (i) is a solution of D.E. $y^2 y' + y^2 + 1 = 0$ ...(ii)

Differentiating both sides of (i), w.r.t. x, $1 + y' = + y'$,

$(1 + y')(1 + y^2) = y' \Rightarrow 1 + y^2 + y' + y'y^2 = y'$

Cross-multiplying

$\Rightarrow y^2y' + y^2 + 1 = 0$ which is same as D.E. (ii). $\therefore$ Function given by (i) is a solution of D.E. (ii).

10. $y =$, $x \in (- a, a) : x + y$

$= 0 \; (y \neq 0)$

Sol. Given: $y =$ [redacted], $x \in (- a, a)$ ...(i) To prove that function given by (i) is a solution of D.E. $x + y$

$= 0$ ...(ii)

From (i),

$= (a^2 - x^2)^{- 1/2} (a_2 - x^2)$

$=$

....(iii)

Putting these values of y and from (i) and (iii) in L.H.S. of (ii),

(– 2x) =

L.H.S. = x + y

= x +

= x – x = 0 = R.H.S. of D.E. (ii).

∴ Function given by (i) is a solution of D.E. (ii).

11. Choose the correct answer:

The number of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0 (B) 2 (C) 3 (D) 4. Sol. Option (D) 4 is the correct answer.

Result. The number of arbitrary constants (c_1, c_2, c_3 etc.) in the general solution of a differential equation of nth order is n. 12. The number of arbitrary constants in the particular solution of a differential equation of third order are (A) 3 (B) 2 (C) 1 (D) 0. Sol. The number of arbitrary constants in a particular solution of a differential equation of any order is zero (0).

[∵ By definition, a particular solution is a solution which contains no arbitrary constant.]

∴ Option (D) is the correct answer.

Exercise 9.3

In each of the Exercises 1 to 5, form a differential equation representing the given family of curves by eliminating arbitrary constants a and b.

1. +
= 1

Sol. Equation of the given family of curves is += 1 ...(i) Here there are two arbitrary constants a and b. So we shall differentiate both sides of (i) two times w.r.t. x.

From (i), . 1 +

= 0 or = –

...(ii)

Again diff. (ii) w.r.t. x, 0 = –

Multiplying both = 0.

sides by – b, Which is

the required D.E.

Remark. We need not eliminate a and b because they have already got eliminated in the process of differentiation. 2. $y^2 = a(b^2 – x^2)$ Sol. Equation of the given family of curves is

$y^2 = a(b^2 - x^2)$ ...(i) Here there are two arbitrary constants a and b.
So, we are to differentiate (i) twice w.r.t. x.

From (i), 2y

= a(0 – 2x) = – 2ax.

Dividing by 2, y

= – ax ...(ii)

Again differentiating both
sides of (ii) w.r.t. x,

y +

. = – a or y

+ = – a ...(iii)

Putting this value of – a from (iii) in (ii), (To eliminate a, as b is already absent in both (ii) and (iii)), we have

x or xy

+

= y

y

= + x

or xy

– y

+ x = 0.

3. $y = ae^{3x} + be^{-2x}$

Sol. Equation of the family of curves is

$y = a\,e^{3x} + b\,e^{-2x}$ ...(i) Here are two arbitrary constants a and b.

From (i),

$= 3\,ae^{3x} - 2\,be^{-2x}$ ...(ii) Again differentiating both sides of (ii), w.r.t. x,

$= 9\,ae^{3x} + 4\,be^{-2x}$ ...(iii)

Let us eliminate a and b from (i), (ii) and (iii).

Equation (ii) – 3 × eqn. (i) gives (To eliminate a),

$- 3y = - 5\ be^{-2x}$...(iv) Again Eqn. (iii) – 3 × eqn (ii) gives (again to eliminate a)

– 3

$= 10\ be^{-2x}$...(v)

Now Eqn. (v) + 2 × eqn. (iv) gives (To eliminate b)

+ 2

$= 10\ be^{-2x} -$

$10\ be^{-2x}$

or

or

– 3

– 3

– 6y = 0

– 6y = 0

+ 2

which is the required D.E.

4. $y = e^{2x}(a + bx)$

Sol. Equation of the given family of curves is

$y = e^{2x}(a + bx)$ ...(i) Here are two arbitrary constants a and b.

=

From (i), or $+ e^{2x}$

(a + bx)

(a + bx)

$= 2\,e^{2x}(a + bx) + e^{2x}.\,b$

or

$= 2y + be^{2x}$ ...(ii) (By (i))

Again differentiating both sides of (ii), w.r.t. x

$= 2$

$+ 2\,be^{2x}$ ...(iii)

Let us eliminate b from eqns. (ii) and (iii), (as a is already absent in both (ii) and (iii))

From eqn. (ii)

$- 2y = be^{2x}$

Putting this value of be^{2x} in (iii), we have

$-$

$\Rightarrow$

$+ 2$

$= 2 \quad = 2$

$+ 2$

$- 4y$

$- 4$ or

$+ 4y = 0$

which is the required D.E.

5. $y = e^{x}(a \cos x + b \sin x)$

Sol. Equation of family of curves is

$y = e^{x}(a \cos x + b \sin x)$ ...(i)

$=$

$\therefore$

(

a
c
o
s
x
+
b
s

i
n
x
)
+
e
x

(– a sin x + b cos x) 11

or

^{x}(a cos x + b sin x) + e

= e

^{x}(– a sin x + b cos x) ...(ii)

$_x$(– a sin x + b cos x)

(By (i))

or

= y + e

Again differentiating both sides of eqn. (ii), w.r.t. x, we have

=

= 2

^{x}(–

^{x}(– a sin x + b cos x) + e

a cos x – b sin x)

+

– – e

x

(a cos x + b sin x)

(By (ii))

+ e

or

=

(By (i))

or

– y – y

– 2

+ 2y = 0 which is the required D.E.

or

6. Form the differential equation of the family of circles touching the

y-axis at the origin.

Sol. Clearly, a circle which touches y-axis at the origin must have its centre on x-axis.

[$\because$ x-axis being at right angles to tangent y-axis is the normal or line of radius of the circle.]

$\therefore$ The centre of circle is (r, 0) where r is the radius of the circle.

$\therefore$ Equation of required circles is

$-\beta)^2 = r^2]$ or $x^2 + r^2 - 2rx +$

$y^2 = r^2$

$(x - r)^2 + (y - 0)^2 = r^2[(x - \alpha)^2 + (y$

or $x^2 + y^2 = 2rx$ ...(i) where r is the only arbitrary constant. $\therefore$

Differentiating both sides of (i) only once w.r.t. x, we have $2x +$

$2y$

$= 2r$ ...(ii)

To eliminate r, putting the value of 2r from (ii) in (i),

$x^2 + y^2 =$

□ + □ x

□ □ □

or $x^2 + y^2 = 2x^2 +$ 2xy 1, 2xy

O X

$+ x_2 - y^2 = 0$

or $-$ 2xy

$- x_2 + y^2 = 0$ or 2xy

r Y

(, 0) r C

Multiplying by $-$

$+ x^2 = y^2$ which is the required D.E.

Remark. The above question can also be stated as : Form the D.E. of the family of circles passing through the origin and having centres on x-axis.

7. Find the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis. Sol. We know that equation of parabolas having vertex at origin and axis along positive y-axis is $x^2 = 4ay$ ...(i) Here a is the only arbitrary constant. So differentiating both sides of

Eqn. (i) only once w.r.t. x, we have

$2x = 4a$

from (i) in (ii),

To eliminate a, putting Y we have

...(ii)

$4a =$

O X

$2x =$

2

x

$\Rightarrow 2xy =$

$\Rightarrow - x$

$+ 2y = 0$

$\Rightarrow x$

(VERTEX)

Dividing both sides by x,

required D.E.

$2y = x$

$- 2y = 0$ which is the

8. Form the differential equation of family of ellipses having foci on y-axis and centre at the origin.

Y

Sol. We know that equation of ellipses having foci on y-axis i.e., vertical ellipses with major axis as y-axis is

Major Axis (0,) a

$+ = 1$ ...(i) F b b Focus

So we shall differentiate eqn.

X′

Here a and b are two arbitrary constants.

(i) twice w.r.t. x.

Differentiating both sides of (i) w.r.t. x, we have

O X

(– , 0) b (, 0) b

F′

2y
Minor Axis Focus
(0, –) a

Y′

+ 2x = 0

or y

Dividing both sides by 2,

= –
x

y –
x ...(ii)

=

Again differentiating both sides of (ii) w.r.t. x, we have

+
–
...(iii)

=

To eliminate a and b, putting this value of

– from (iii) in (ii),

the required differential equation is

=

+ x

Multiplying both

y

sides by a^2, y

+ x

=

xy

+ x

or xy − y

= 0

which is the required differential equation.

9. Form the differential equation of the family of hyperbolas having foci on x-axis and centre at the origin.

Y

Conjugate Axis

(– , 0) a

F (Focus) 1

Sol. We know that equation of F hyperbolas having foci on x axis and centre at origin is

O X

2 Transverse

(, 0) a

Axis

(Focus)

− = 1 ...(i)

Here a and b are two arbitrary constants. So we shall differ entiate eqn. (i) twice w.r.t. x.

From (i), . 2x − . 2y

= 0 or x = y

Dividing both sides by 2, x = y

...(ii)

Again differentiating both sides of (ii), w.r.t. x,

. 1 = .

+

or =

+ ...(iii)

Dividing eqn. (iii) by eqn. (ii), we have (To eliminate a and b)

+

=

+

Cross-multiplying, x = y

or xy

+ x = 0

$-$ y

which is the required differential equation.

10. Form the differential equation of the family of circles having centres on y-axis and radius 3 units.

Sol. We know that on y-axis, x = 0.

$\therefore$ Centre of the circle on y-axis is (0, β).

$\therefore$ Equation of the circle having centre on y-axis and radius 3 units is

$(x - 0)^2 + (y - \beta)^2 = 3^2 [(x - \alpha)^2 + (y - \beta)^2 = r^2]$ or $x^2 + (y - \beta)^2 = 9$ ...(i)

Here β is the only arbitrary constant. So we shall differentiate both sides of eqn. (i) only once w.r.t. x,

From (i), 2x + 2 (y – β) (y – β) = 0

or 2x + 2 (y – β)

= 0

–
=
–

or 2 (y – β)

= – 2x ∴ y – β =

Putting this value of (y – β) in (i) (To eliminate β), we have

x^2 +

= 9

sides by this L.C.M.,

+ x_2 = 9

L.C.M. =

. Multiplying

both

x^2

– 9

– 9)

+ x_2 = 0 or (x^2

⇒ x^2 + x_2 = 0

which is the required differential equation.

x + c_2 e^{-x} as the

11. Which of the following differential equation has y = c_1 e general solution?

(A)

– y = 0

+ y = 0 (B)

– 1 = 0

(C)

+ 1 = 0 (D) $^{x} + c_2 e^{-x}$...(i) ∴

Sol. Given: $y = c_1 e$

$^{x} + c_2 e^{-x}(-1) = c_1 e$

$= c_1 e$ $^{x} - c_2 e^{-x}$

$^{x} - c_2 e^{-x}(-1) = c_1 e$

$^{x} + c_2 e^{-x}$ $= c_1 e$

∴

or

= y [By (i)]

or

– y = 0 which is differential equation given in option (B)

∴ Option (B) is the correct answer.

12. Which of the following differential equations has y = x as one of its particular solutions?

(C)

(A) $-x^2$ + x (D)

$-x^2$ + x (B)

+ xy = 0

+ xy = x

+ xy = 0

+ xy = x

Sol. Given:

y = x ∴ = 0

= 1 and

These values of y, and $\frac{dy}{dx}$ clearly satisfy the D.E. of option (C).

[∵ L.H.S. of D.E. of option (C) = $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy$

$= 0 - x^2(1) + x(x) = -x^2 + x^2 = 0$ = R.H.S. of option (C)] ∴ Option (C) is the correct answer.

Exercise 9.4 *(Page No. 395-397)*

For each of the differential equations in Exercises 1 to 4, find the general solution:

1. $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

Sol. The given differential equation is $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

or $dy = \frac{1-\cos x}{1+\cos x}\,dx$.

Integrating both sides,

$\int dy = \int \frac{1-\cos x}{1+\cos x}\,dx$

or y = $\int$ dx =

– x + c

Exercise 9.4

 – $\int$ dx =

For each of the differential equations in Exercises 1 to 4, find the general solution:

=

1.

Sol. The given differential equation is =

or dy =

+

– –

+

dx.

sides,

Integrating both $\int$ dx

$\int =$ $\left(- \right) \int dx =$

or y = $\int dx =$

– x + c

or y = 2 tan – x + c

which is the required general solution.

2.

= (– 2 < y < 2)

Sol. The given D.E. is

$= - \Rightarrow dy = - dx$

Separating variables, – = dx

Integrating both sides, –

$\int dy = \int dx$

$\therefore \sin^{-1} = x + c$

$\because \int$

$=$

$\Rightarrow$ = sin (x + c)

$\Rightarrow$ y = 2 sin (x + c) which is the required general solution.

3.

+ y = 1 (y ≠ 1)

Sol. The given differential equation is

+ y = 1

$\Rightarrow$

$= 1 - y \Rightarrow dy = (1 - y)\, dx \Rightarrow dy = -(y - 1)\, dx$

Separating variables,

$= -\,dx$

dx $\int = -\int$

Integrating both sides,

$\Rightarrow \log|y-1| = -x + c$

$\Rightarrow |y-1| = e^{-x+c}$ [$\because$ If $\log x = t$, then $x = e^t$] $\Rightarrow y - 1 = \pm e^{-x+c} \Rightarrow y = 1 \pm e^{-x} e^c$

$\Rightarrow y = 1 \pm e^c e^{-x}$

$\Rightarrow y = 1 + Ae^{-x}$ where $A = \pm e^c$

which is the required general solution.

4. $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

Sol. The given differential equation is

$$\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$$

Dividing by tan x tan y, we have

dx + dy = 0 (Variables separated)

Integrating both sides, $\int$ dx + $\int$ dy = log c

or $\log|\tan x| + \log|\tan y| = \log c$

$\because \int$

or $\log|(\tan x \tan y)| = \log c$ or $|\tan x \tan y| = c$ $\therefore \tan x \tan y = \pm c = C$ where $C = \pm c$.

[$\because |t| = a (a \geq 0) \Rightarrow t = \pm a$]

which is the required general solution.

For each of the differential equations in Exercises 5 to 7, find

the general solution:
$^{x} + e^{-x}$) dy – (e
5. (e $^{x} - e^{-x}$) dx
$^{x} - e^{-x}$) dx = 0 ^{x} + dx
e^{-x}) dy = (e
Sol. The given
D.E. is (e

or dy =
Integrating both sides, $\int$ =

+ $\int$ dx

$^{x} + e^{-x}$| + c
or y = log | e
$\because$ $\int$

which is the required general solution.

6.
= $(1 + x^2)(1 + y^2)$

Sol. The given differential equation is $= (1 + x^2)(1 + y^2)$

$\Rightarrow$ dy = $(1 + x^2)(1 + y^2)$ dx
Separating variables, + = $(1 + x^2)$ dx

Integrating both sides,
c
+ $\int$
dy = + $\int$
dx $\Rightarrow$ $\tan^{-1} y$ = + x +

which is the required general solution.
7. y log y dx – x dy = 0
Sol. The given differential equation is y log y dx – x dy = 0 $\Rightarrow$ – x dy = – y log y dx

= ...(i)

Separating variables,

$\int = \int$

Integrating both sides

For integral on left hand side, put log y = t.

dt

$\therefore$

$= \Rightarrow =$

$\therefore$ Eqn. (i) becomes $\int = \int$

$\Rightarrow$ log | t | = log | x | + log | c |*...(ii) = log | xc |

$\Rightarrow$ | t | = | xc |

$\Rightarrow$ t = ± xc

[$\because$ | x | = | y | $\Rightarrow$ x = ± y]

$\Rightarrow$ log y = ± xc = ax where a = ± c

$\therefore y = e^{ax}$ which is the required general solution.

For each of the differential equations in Exercises 8 to 10, find the general solution:

8. $x^5 = -y^5$

Sol. The given differential equation is x^5

$\Rightarrow x^5\, dy = -y^5\, dx$

$= -y^5$

Separating variables,

$\Rightarrow y^{-5}\, dy = -x^{-5}\, dx$

$= -$

Integrating both sides,

$\int$ dy = – $\int$ dx

$= - + c$

Multiplying by – 4, $y^{-4} = -x^{-4} - 4c$

$\Rightarrow x^{-4} + y^{-4} = -4c \Rightarrow x^{-4} + y^{-4} = C$ where C = – 4c which is the required general solution.

9.

$= \sin^{-1} x$

Sol. The given differential equation is
or $dy = \sin^{-1} x \, dx$
$= \sin_{-1} x$

$dy = -$

Integrating both
sides, $\int$

$\int$
dx

or $y = -$
$\int . 1 \, dx$

I II
Applying product
rule,

$\int = x \sin^{-1} x$

$\int (\sin_{-1} x)$

$-$

dx –

$\int$

dx dx

$y = (\sin^{-1} x)$

$- \int$

dx = –

x dx ...(i)

To evaluate – $\int$

dx

$-$

$- \int$

Put $1 - x^2 = t$. Differentiate $-2x \, dx = dt$

*Remark. To explain *in eqn. (ii)

If all the terms in the solution of a D.E. involve logs, it is better to use log c or log | c | instead of c in the solution.

$- \int dx = - \int = -$

$\therefore$

$\int$ dt

$= -$

$= -$

$= - -$

Putting this value of $- \int dx$ in (i), the required general solution is

$$y = x \sin^{-1} x + [redacted] + c.$$

10. $e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$

Sol. The given equation is $e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$

Dividing every term by $(1 - e^x) \tan y$, we have

$- dx +$

$dy = 0$

(Variables separated)

Integrating both sides, $- \int dx + \int dy = c$

$- \int dx + \log | \tan y | = c$

or $- \log | 1 - e^x | + \log | \tan y | = c$

$\because \int$ ′

or log

$- = \log c'$ (See Remark at the end of page 612) or

$- = c'$

or $\tan y = C (1 - e^{x})$. [$\because$ | t | = c′ ⇒ t = ± c′ = C (say)] For each of the differential equations in Exercises 11 to 12, find a particular solution satisfying the given condition: 11. $(x^3 + x^2 + x + 1)$

= $2x^2 + x$, y = 1, when x = 0

Sol. The given differential equation is $(x^3 + x^2 + x + 1)$

$= 2x^2 + x$

$\therefore (x^3 + x^2 + x + 1)\ dy = (2x^2 + x)$
dx

variables dy =
+
dx
+ ++
Separating
+

or dy =

dx
+ +

[$\because$
$x^3 + x^2 + x + 1 = x^2(x + 1) + (x + 1) = (x + 1)(x^2 + 1)$]
Integrating both sides, we have

+ +$\int$ dx or y =
+ +
dy =

$\int$

!

Let

$+ + \int dx$ ...(i)

$=$ fractions)

$+ +$

$+ + + +$(Partial

...(ii)

Multiplying both sides by L.C.M. = $(x + 1)(x^2 + 1)$, we have $2x^2 +$

$x = A(x^2 + 1) + (Bx + C)(x + 1)$

or $2x^2 + x = Ax^2 + A + Bx^2 + Bx + Cx + C$

Comparing coeff. of x^2 on both sides, we have

$A + B = 2$ ...(iii)

Comparing coeff. of x on both sides, we have

$B + C = 1$ ...(iv)

Comparing constants $A + C = 0$ ...(v) Let us solve eqns. (iii), (iv) and (v) for A, B, C eqn. (iii) – eqn. (iv) gives to eliminate B, $A - C = 1$ ...(vi)

Adding (v) and (vi), $2A = 1$ or $A =$

From (v), $C = - A = -$

Putting $C = -$ in (iv), $B - = 1$ or $B = 1 + =$ Putting these values of A, B, C in (ii), we have

$-$

$+$

$+ + = + + +$

$+$

$=$

$+ -$ $. +$ $=$

$+$ $+ -$

$+$ $+$

$+$

.

Putting this value in (i)

$+ \int dx +$

$+\int dx -$

$+\int dx$

y =

$y = \log(x+1) + \log(x^2+1) - \tan^{-1} x + c$ ...(vii)

$\because \int$

To find c

When x = 0, y = 1 (given)

Putting x = 0 and y = 1 in (vii),

$1 = \log 1 + \log 1 - \tan^{-1} 0 + c$

or 1 = c [$\because$ log 1 = 0 and $\tan^{-1} 0 = 0$] Putting c = 1 in eqn. (vii), the required solution is

$$y = \log(x+1) + \log(x^2+1) - \tan^{-1} x + 1.$$

$$y = [2\log(x+1) + 3\log(x^2+1)] - \tan^{-1} x + 1 = [\log (x+1)^2 + \log(x^2+1)^3] - \tan^{-1} x + 1$$

$$= [\log (x+1)^2(x^2+1)^3] - \tan^{-1} x + 1$$

which is the required particular solution.

12. $x(x^2 - 1)$

= 1; y = 0 when x = 2.

Sol. The given differential equation is $x(x^2 - 1) = 1$

$\Rightarrow x(x^2-1)\, dy = dx \Rightarrow \int \Rightarrow y =$

dy =

$-$ dy =

Integrating both sides, $-\int dx$

dx + c ...(i) Let

$+ - \int$ the integrand =

$+ -$

+ +!

_...(ii)

(By Partial Fractions)

Multiplying by L.C.M. = x(x + 1)(x – 1),

+

1 = A(x + 1)(x – 1) + Bx(x – 1) + Cx(x + 1)

or $1 = A(x^2 - 1) + B(x^2 - x) + C(x^2 + x)$

or $1 = Ax^2 - A + Bx^2 - Bx + Cx^2 + Cx$

Comparing coefficients of x^2, x and constant terms on both sides, we have

x^2: A + B + C = 0 ...(iii) x: – B + C = 0 ⇒ C = B ...(iv) Constants – A = 1 or A = – 1

Putting A = – 1 and C = B from (iv) in (iii),

– 1 + B + B = 0 or 2B = 1 ⇒ B =

∴ From (iv), C = B =

Putting these values of A, B, C in (ii),

–

+ –

= – +

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+ +

∴

+ – $\int$ dx = – $\int$ dx +

+ $\int$ dx +

– $\int$ dx

= – log | x | + log | x + 1 | + log | x – 1 |

= [– 2 log | x | + log | x + 1 | + log | x – 1 |]

= [– log | x |^2 + log | (x + 1)(x – 1) |]

+ – $\int$ dx =

⇒

Putting this value in (i), – – =

$y = \log -$

$+ c$ ...(v)

To find c for the particular solution

Putting y = 0, when x = 2 (given) in (v),

$0 = \log + c \Rightarrow c = -\log$

Putting this value of c in (v), the required particular solution is y =

log

$- \log$

OR

dx =

$-\int$

To evaluate

dx =

$-\int$

$-\int$

dx

Put $x^2 = t$.

For each of the differential equations in Exercises 13 to 14, find a particular solution satisfying the given condition:

13. cos

= a (a ∈ R); y = 1 when x = 0

Sol. The given differential equation is

cos

= a (a ∈ R); y = 1 when x = 0

∴

$= \cos^{-1} a \Rightarrow dy = (\cos^{-1} a)\, dx$

Integrating both sides

$\int dy = \int (\cos^{-1} a) \, dx \Rightarrow y = (\cos^{-1} a) \int dx$

$\Rightarrow y = (\cos^{-1} a)\, x + c$ ...(i) To find c for particular solution y = 1 when x = 0 (given) $\therefore$ From (i), 1 = c.

Putting c = 1 in (i), $y = x \cos^{-1} a + 1$

$\Rightarrow y - 1 = x \cos^{-1} a \Rightarrow \frac{y-1}{x} = \cos^{-1} a$

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$\Rightarrow \cos\left(\frac{y-1}{x}\right) = a$ which is the required particular solution.

14. $\frac{dy}{dx}$ = y tan x; y = 1 when x = 0

Sol. The given differential equation is $\frac{dy}{dx} = y \tan x$

$\Rightarrow dy = y \tan x \, dx$

Separating variables, $\frac{dy}{y} = \tan x \, dx$

Integrating both sides $\int \frac{dy}{y} = \int \tan x \, dx \Rightarrow \log |y| = \log |\sec x| + \log |c|$

$\Rightarrow \log |y| = \log |c \sec x| \Rightarrow |y| = |c \sec x| \therefore y = \pm c \sec x$ or $y = C \sec x$ ...(i) where $C = \pm c$

To find C for particular solution

Putting y = 1 and x = 0 in (i), 1 = C sec 0 = C

Putting C = 1 in (i), the required particular solution is y = sec x. 15. Find the equation of a curve passing through the point (0, 0) and whose ^{x}sin x.

differential equation is $y' = e$

$^x\sin x$

Sol. The given differential equation is $y' = e$

⇒

$^x\sin x$ ⇒ $dy = e$

$= e$ $^x\sin x\,dx$

$dy = \int$ dx

Integrating

both sides, $\int$

= − $\int \int\int\int$

or $y = I + C$ ...(i) where $I = \int$

dx ...(ii) I II

"" # $ % & && & &&

dx

I II

& && $^x(-\cos x) -$

Again applying

$-\int$

product rule, $^x\sin x$

$= e$

$- \int$

$^x\cos x + e$

dx

$I = -e$

$^x\cos x + \int$

dx

⇒ $I = -e$

$^x(-\cos x + \sin x) - I$ [By (ii)] Transposing $2I = e$

∴ $I =$

⇒ $I = e$ $^x(\sin x - \cos x)$

$(\sin x - \cos x)$

Putting this value of I in (i), the required solution is

$^x(\sin x - \cos x) + c$ ...(iii) To find c. Given that required

$y = e$

curve (i) passes through the point (0, 0).

Putting $x = 0$ and $y = 0$ in (iii),

$0 = (-1) + c$ or $0 = -$ $+ c$ ∴ $c =$

in (iii), the required equation of the curve is

Putting $c =$ $y = e$ ^{x}

$(\sin x - \cos x) +$

$^x(\sin x - \cos x) + 1$ or $2y - 1 = e$

L.C.M. = 2 ∴ $2y = e$

which is the required equation of the

curve.

16. For the differential equation xy = (x + 2)(y + 2), find

^{x}(sin x – cos x)

the solution curve passing through the point (1, – 1). Sol.

The given differential equation is
xy = (x + 2)(y + 2)

⇒ xy dy = (x + 2)(y + 2) dx

\+ dy =

Separating variables

\+ ∫ dx

\+

dx

\+ ∫ dy =

Integrating both sides,

⇒ + –

\+ ∫ dy =

 ∫ dx

 +

⇒

\+

  –

  + + ∫ dy =

 +   ∫ dx

⇒

 –

  + ∫ dy =

 +

  ∫ dx

⇒ y – 2 log | y + 2 | = x + 2 log | x | + c

⇒ $y - x = \log(y+2)^2 + \log x^2 + c$ | ∵ $|x|^2 = x^2$ ⇒ $y - x = \log((y+2)^2 x^2)$

+ c ...(i) To find c. Curve (i) passes through the point (1, – 1).

Putting x = 1 and y = – 1 in (i), – 1 – 1 = log (1) + c or – 2 = c ($\because$ log 1 = 0)

Putting c = – 2 in (i), the particular solution curve is y – x = log ((y + 2)^2 x^2) – 2

or y – x + 2 = log ((y + 2)^2 x^2).

17. Find the equation of the curve passing through the point (0, – 2) given that at any point (x, y) on the curve the product of the slope of its tangent and y-coordinate of the point is equal to the x-coordinate of the
the
point.

Sol. Let P(x, y) be any point on the required curve.

According to the question,

(Slope of the tangent to the curve at P(x, y)) × y = x

⇒

. y = x ⇒ y dy = x dx

Now variables are separated.

Integrating both sides $\int$ dy = $\int$ dx ∴ = + c Multiplying by L.C.M. = 2, y^2 = x^2 + 2c

or y^2 = x^2 + A ...(i) where A = 2c.

Given: Curve (i) passes through the point (0, – 2).

Putting x = 0 and y = – 2 in (i), 4 = A.

Putting A = 4 in (i), equation of required curve is

y^2 = x^2 + 4 or y^2 – x^2 = 4.

18. At any point (x, y) of a curve the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (– 4, – 3). Find the equation of the curve given that it passes through (– 2, 1).

Sol. According to question, slope of the tangent at any point P(x, y) of the required curve.

= 2 . (Slope of the line joining the point of contact P(x, y) to the given point A(– 4, – 3)).

– –

= 2 ⇒

=

– – –

+

+
−

⇒
Cross-multiplying, $(x + 4)\, dy = 2(y + 3)\, dx$

+ =

Separating variables,

Integrating both

sides, $+ \int dy = 2$
$+ \int dx$
+ dx
$\Rightarrow \log|y + 3| = 2\log|x + 4| + \log|c|$
(For $\log|c|$, see Foot Note page 612)
$\Rightarrow \log|y + 3| = \log|x + 4|^2 + \log|c| = \log|c|(x + 4)^2 \Rightarrow |y + 3| = |c|(x + 4)^2$
$\Rightarrow y + 3 = \pm|c|(x + 4)^2$
$\Rightarrow y + 3 = C(x + 4)^2$ ...(i) where $C = \pm|c|$ 27

To find C. Given that curve (i) passes through the point (– 2, 1).
Putting $x = -2$ and $y = 1$ in (i),

$$1 + 3 = C(-2 + 4)^2 \text{ or } 4 = 4C \Rightarrow C == 1.$$

Putting C = 1 in (i), equation of required curve is

$$y + 3 = (x + 4)^2 \text{ or } (x + 4)^2 = y + 3.$$

19. The volume of a spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

Sol. Let x be the radius of the spherical balloon at time t. Given: Rate of change of volume of spherical balloon is constant = k (say) π =

$k \Rightarrow \pi\, 3x^2 = k \Rightarrow 4\pi x^2 = k$

⇒
Separating variables, $4\pi x^2\, dx = k\, dt$

Integrating both sides, $4\pi \Rightarrow 4\pi \int$ $dx = k \int$ $= kt + c$ ...(i) dt

To find c: Given: Initially radius is 3 units. ⇒

When t = 0, x = 3

Putting t = 0 and x = 3 in (i), we have

$\pi(27) = c$ or $c = 36\pi$ ...(ii)

To find k: Given: When t = 3 sec, x = 6 units

Putting t = 3 and x = 6 in (i), $\pi(6)^3 = 3k + c$.

Putting $c = 36\pi$ from (ii), $\pi(216) = 3k + 36\pi$

or $4\pi(72) - 36\pi = 3k \Rightarrow 288\pi - 36\pi = 3k$ or $3k = 252\pi \Rightarrow k = 84\pi$ ...(iii)

Putting values of c and k from (ii) and (iii) in (i), we have π + 36π $x^3 = 84\pi t$

Dividing both sides by 4π, = 21t + 9

$\Rightarrow x^3 = 63t + 27 \Rightarrow x = (63t + 27)^{1/3}$.

20. In a bank principal increases at the rate of r % per year. Find the value of r if ` 100 double itself in 10 years. ($\log_e 2 = 0.6931$)

Sol. Let P be the principal (amount) at the end of t years.

According to given, rate of increase of principal per year = r% (of the principal)

⇒ $ = " × P

Separating variables, $ = " dt

Integrating both sides, log P = " $ is

t + c ...(i) (Clearly P being principal > 0, and hence log | P | = log P) To find c. Initial principal = ` 100 (given)

i.e., When t = 0, P = 100

Putting t = 0 and P = 100 in (i), log 100 = c.

Putting c = log 100 in (i), log P = " t + log 100

⇒ log P – log 100 = " t ⇒ log $ t ...(ii)

" = "

Putting P = double of itself = 2 × 100 = ` 200

When t = 10 years (given) in (ii),

log " = × 10 ⇒ log 2 = ,
" "

⇒ r = 10 log 2 = 10 (0.6931) = 6.931% (given).

21. In a bank, principal increases at the rate of 5% per year. An amount of ` 1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5}$ = 1.648).

Sol. Let P be the principal (amount) at the end of t years.

According to given rate of increase of principal per year = 5% (of the principal)

⇒ $

= " × P ⇒ $

⇒ 20 dP = P dt $,

=

Separating variables, = '

$ $

Integrating both sides, we have

log P = ' t + c ...(i) To find c. Given: Initial principal deposited with the bank is ` 1000.

⇒ When t = 0, P = 1000

Putting t = 0 and P = 1000 in (i), we have log 1000 = c Putting c = log 1000 in (i), log P = ' + log 1000 ⇒ log P – log 1000 = , ⇒

log $

"' = ,

Putting t = 10 years (given), we have

log $

"' = '

' = = 0.5

⇒ $

''' = $e^{0.5}$ [∵ If log x = t, then x = e

^{t}] ⇒ P = 1000 $e^{0.5}$ = 1000 (1.648)

[∵ ` 1648.
'''
$e^{0.8}$ = 1.648 (given)]

= 1000( )

22. In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present.

Sol. Let x be the bacteria present in the culture at time t hours. According to given,

Rate of growth of bacteria is proportional to the number present. is proportional to x.

∴ i.e.,

= kx where k is the constant of proportionality (k > 0 because rate of growth (i.e., increase) of bacteria is given.)

⇒ dx = kx dt ⇒ = k dt

Integrating both sides, $\int$ dx = k $\int$ dt

⇒ log x = kt + c ...(i) To find c. Given: Initially the bacteria count is x_0 (say) = 1,00,000.

⇒ When t = 0, x = x_0.

Putting these value in (i), log x_0 = c.

Putting c = log x_0 in (i), log x = kt + log x_0

⇒ log x – log x_0 = kt ⇒ log = kt ...(ii) To find k: According to given, the number of bacteria is increased by 10% in 2 hours.

∴ Increase in bacteria in 2 hours = '

'' × 1,00,000 = 10,000

∴ x, the amount of bacteria at t = 2

= 1,00,000 + 10,000 = 1,10,000 = x_1 (say)

Putting x = x_1 and t = 2 in (ii),

= 2k ⇒ k = log '

log '

⇒ k = log* '*''

=

''''

log '

 Putting this value of k in (ii),

we have log ' = '

t

When x = 2,00,000 (given);

then log *''*''

*

''*'' = '

t ⇒ log 2 = log '

t

t ⇒ t =

Cross-multiplying 2 log 2 = '

hours. '

23. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(A) $e^x + e^{-y} = c$ (B) $e^x + e^y = c$ (C) $e^{-x} + e^y = c$ (D) $e^{-x} + e^{-y} = c$

Sol. The given D.E. is $\frac{dy}{dx} = e^{x+y}$ ⇒

$^{y} \Rightarrow dy = {}^{y} dx$
e

^{x}. e

= e
^{x}. e $\qquad ^{x}$ dx or e^{-y} dy = e

^{x} dx

= e

Separating variables,

Integrating both sides^{-}

dx $\int$ dy = $\int$

–

^{x} + c $\Rightarrow -e^{-y} - e$

^{x} = c $\qquad ^{x} = -c$

$\Rightarrow$

_ = e

Dividing by – 1, e^{-y} + e

^{x} + e^{-y} = C where C = – c which is the required solution. ∴

or e

Option (A) is the correct answer.

Exercise 9.5

In each of the Exercises 1 to 5, show that the given differential equation is homogeneous and solve each of them:

1. $(x^2 + xy)\, dy = (x^2 + y^2)\, dx$

Sol. The given D.E. is

$(x^2 + xy)\, dy = (x^2 + y^2)\, dx$ ...(i) This D.E. looks to be homogeneous as degree of each coefficient of dx and dy is same throughout (here 2).

+

From (i),

= +

+ =

+ +

or

=

...(ii)

+

= F

∴ The given D.E. is homogeneous.

Put = v. Therefore y = vx.

∴

= v . 1 + x

= v + x

Putting these values of and

in (ii), we have

v + x =

+

+

+

Transposing v to
R.H.S., x

+ =

=

⇒ x =

+ −−

−

+

+− v

Cross-multiplying x(1 + v) dv = (1 – v) dx

Separating variables

+

dv =

−

Integrating both sides

+

$-\int$ dv = $\int$

⇒

$-\int$ dv = log x + c ⇒

$-\int$ dv = log x + c

−

⇒

+−+ − −

− − v = log x + c , − 2

− $\int$ dv = log x + c ⇒

⇒ − 2 log (1 − v) − v = log x + c Put v =

log

− = log x + c

− + = − log x − c

Dividing by − 1, 2 log

− + log x = − − c ⇒ log −

⇒ log = − − c

− = − − = − e

− c ⇒ $(x-y)^2 = Cx^-$ where C = e − c

⇒

which is the required solution.

2. y′ =

+

Sol. The given differential equation is y′ =

$\Rightarrow$ = 1 + = f

= + $\Rightarrow$
$\therefore$ Differential equation (i) is homogeneous.

...(i)

Put = v $\therefore$ y = vx

$\therefore$

= v . 1 + x

= v + x

Putting these values of

and y in (i),

v + x

= 1 + v $\Rightarrow$ x

= 1 $\Rightarrow$ x dv = dx

Separating variables, dv =

v = ,

Integrating both sides, $\int$

dv = $\int$ v = log | x | + c Putting

= log | x | + c $\therefore$ y = x log | x | + cx which is the required solution. 3. (x – y) dy – (x + y) dx = 0

Sol. The given differential equation is

(x – y) dy – (x + y) dx = 0 ...(i) Differential equation (i) looks to be homogeneous because each coefficient of dx and dy is of degree 1.

From (i), (x – y) dy = (x + y) dx

+

+

=

=

$\therefore$

+

− or − = f =

∴ Differential equation (i) is

homogeneous. Put = v ∴ y =

vx ∴

= v . 1 + x

= v + x

+ x

=

Shifting v to R.H.S., x

+

=

+

Putting these values in (ii), v −

...(ii)

− v =

+−+

−

+ −

⇒ x =

Cross-multiplying, x (1 – v) dv = (1 + v^2) dx −

Separating variables,

+ dv =

−

Integrating both sides,

+∫ dv = ∫ dx + c

∫ dv –

+

∫ dv = ∫ dx + c

⇒

⇒ $\tan^{-1}$

v –
+

∫ dv = log x + c

Putting v = , tan $^{-1}$ – log

v – log (1 + v^2) = log x + c ∵ ∫

⇒ $\tan^{-1}$

+

= log x + c

⇒ $\tan^{-1}$ – log

+

= log x + c

⇒ $\tan^{-1}$ – [log (x^2 + y^2) – log x^2] = log x + c ⇒ $\tan^{-1}$ – log (x^2 + y^2) +2 log x = log x + c ⇒ $\tan^{-1}$ – log (x^2 + y^2) = c ⇒ $\tan^{-1}$ = log (x^2 + y^2) + c which is the required solution.

4. $(x^2 - y^2)\,dx + 2\,xy\,dy = 0$

Sol. The given differential equation is

$(x^2 - y^2)\,dx + 2xy\,dy = 0$ ...(i) This differential equation looks to be homogeneous because degree of each coefficient of dx and dy is same (here 2).

From (i), $2xy\,dy = -(x^2 - y^2)\,dx$

– – =

⇒

–

=

Dividing every term in the numerator and denominator of R.H.S. by x^2,

– =

f

=

...(ii)

∴ The given differential equation is homogeneous. Put

= v. Therefore y = vx ∴

= v . 1 + x

= v + x

Putting these values of and
we have

in differential equation (ii), v + x =

–
⇒ x – – v =

=
– –

⇒ x
– –
= –

+
$\therefore x\, 2v\, dv = -(v^2 + 1)\, dx$

=

+ = –

$\Rightarrow$

Integrating both sides,

$+\int dv = -\int dx$

$\Rightarrow \log (v^2 + 1) = -\log x + \log c \Rightarrow \log (v^2 +$
$1) + \log x = \log c$

$\Rightarrow \log (v^2 + 1)\, x = \log c$

$\Rightarrow (v^2 + 1)\, x = c$

Put v = , +

x = c or

or

$+$ $+$ = c or $x^2 + y^2 = cx$

x = c

which is the required solution.

5. x^2

$= x^2 - 2y^2 + xy$

Sol. The given differential equation is x^2

$= x^2 - 2y^2 + xy$

The given differential equation looks to be Homogeneous as all terms in x and y are of same degree (here 2).

by x^2,

or

= – +

= 1 – 2

Dividing

+

...(i)

= F

∴ Differential equation (i) is homogeneous.

So put = v ∴ y = vx

∴

= v . 1 + x

= v + x

Putting these values of and

in (i),

v + x

= 1 – 2v2 + v or x

= 1 – 2v2 ⇒ x dv = $(1 - 2v^2)$ dx

–

=

Separating variables,

–

$\int$ dv = $\int$ dx

Integrating both sides,

+

$\Rightarrow$

–

$\rightarrow$ = log | x | + c

!

$\because$ $\int$

Putting v = ,

log

+ – = log | x | + c

Multiplying within logs by x in L.H.S.,

+

log

– = log | x | + c.

In each of the Exercises 6 to 10, show that the given D.E. is homogeneous and solve each of them:

6. x dy – y dx = dx

Sol. The given differential equation is

x dy – y dx = + dx or x dy = y dx + . dx Dividing by dx

x

= y + + or x + = F Dividing by x,

+

= y + x ...(i)

= +

$\therefore$ Given differential equation is homogeneous. Put

= v i.e., y = vx.

Differentiating w.r.t. x,

= v + x

Putting these values of and

in (i), it becomes

v + x

= v + +

or x

= +

∴ x dv = [redacted]

dx or

[redacted]

= Integrating both sides,

[redacted]

$= \int$

$\int$

∴ log (v + [redacted]

) = log x + log c

Replacing v by , we have

□ + + □ □ □ □ = log cx or

□ □

log + + = cx

or y + + = cx^2

which is the required solution.

y dx =

y dx =

7.□ □ □ □ □□ □□ □ □ □□ □□ Sol. □ □ □ □ □ □ + □□ □□ □ □ □□ □□

The given D.E. is

□ □ □ □ □□ □□□ □ □ □ □□ □□ x dy □ □ □ □ □ □ − □□ □□ □ □

$+$

$\therefore$ $+$ $-$ $=$ x dy

$=$

$-$

$-$

$+$

Dividing every term in R.H.S. by x^2, $=$ $= F$ ...(i)

$\therefore$ The given differential equation is homogeneous. So

let us put $=$ v. Therefore $y = vx$.

$\therefore$

$= v \cdot 1 + x$

$= v + x$

Putting these values in differential equation (i), we have

$v + x$ $+$

$-$ $\Rightarrow$ x

$=$ $+$ $= - - v$

+−+

− ⇒ x =

−

=

Cross-multiplying, x(v sin v – cos v) dv = 2v cos v dx

Separating variables,

− dv = 2

Integrating both sides,

−∫ dv = 2 ∫ dx

− = − , ⇒

Using

−

∫ dv = 2 ∫ dx

⇒

−

∫ dv = 2 ∫ dx

⇒ log | sec v | – log | v | = 2 log | x | + log | c |

$\Rightarrow$ log

= log | x |^2 + log | c | = log (| c | x^2)

$\Rightarrow$

= | c | x^2 $\Rightarrow$

= ± | c | x^2

$\Rightarrow$ sec v = ± | c | x^2 v

Putting v = , sec = Cx^2 where C = ± | c | or

sec = Cxy $\Rightarrow$

= Cxy

$\Rightarrow$ C xy cos = 1 $\Rightarrow$ xy cos = ! = C_1 (say) which is the required solution.

8. x – y + x sin = 0

Sol. The given D.E. is x = 0

– y + x sin

= y – x sin

= – sin or x

Dividing every term by x, =

Putting = v i.e., y = vx so that

F

= F

Since homogeneous.

, the given differential equation is

Putting these values of and

v + x = v + x

= v – sin v

or x ...(i) in (i), we have

= – sin v ∴ x dv = – sin v dx– or cosec v dv = – or

=

Integrating, log | cosec v – cot v | = – log | x | + log | c | or log

| cosec v – cot v | = log

or cosec v – cot v = ±

Replacing v by , cosec – cot = ! where C = ± c

⇒ – ! = ⇒ = !

–

Cross-multiplying, x solution. required

is the = C sin which

9. y dx + x log

dy – 2x dy = 0

Sol. The given differential equation is y dx + x

dy – 2x dy = 0

∴ y dx = 2x dy – x

∴

= F

–

= F

dy or y dx = x

...(i)

=

dy

differential Putting = v i.e.,

y = vx so that

equation is

Since homogeneous.

, the given

= v + x

Putting these values of and

in (i), we have

v + x

=

–

– v =

=

–

or x

– +

– +

= or x –

–

=–

– –

–

∴ x(2 – log v) dv = v
(log v – 1) dx

or

dv =

–

or dv =

–

–

–

dv =

or

Integrating

– $\int$ dv = log | x | + log | c |

or log | log v – 1 | – log | v | = log | x | + log | c |

∵ ∫

or log

–
= log | cx | or

–= | cx |
or

–
= ± cx = Cx where C = ± c

or log v – 1 = Cx v

Replacing v by , we have

log – 1 = Cx

or log – 1 = Cy

which is a primitive (solution) of the given differential equation.

Second solution

The given D.E. is y dx + x log $\frac{y}{x}$ dy – 2x dy = 0

Dividing every term by dy,

$$y\frac{dx}{dy} - x\log\frac{x}{y} - 2x = 0$$

log log – log – log – log – log ( ) ==

∵ $\frac{y}{x}$ yx xy

x y

Dividing every term by y,

$$\frac{dy}{dx} - \frac{x}{y}\log\frac{x}{y} - 2\frac{x}{y} = 0$$

$$\Rightarrow \frac{dx}{dy} = \frac{x}{y}\log\frac{x}{y} + 2\frac{x}{y} \quad \ldots \text{(i)} = F\left(\frac{x}{y}\right)$$

∴ The given differential is homogeneous.

Put $\frac{x}{y} = v$ i.e. $x = vy$

so that $\frac{dx}{dy} = v + y\frac{dv}{dy}$

Putting these values in D. E. (i), we have

$$v + y\frac{dv}{dy} = v\log v + 2v$$

$$\Rightarrow y\frac{dv}{dy} = v\log v + v = v(\log v + 1)$$

Cross-multiplying $y\,dv = v(\log v + 1)\,dy$

Separating variables $\frac{dv}{v(\log v + 1)} = \frac{dy}{y}$

Integrating both sides $\int \frac{\frac{1}{v}}{\log v + 1}\,dv = \int \frac{dy}{y}$ $\quad \left[\because \int \frac{f'(v)}{f(v)}\,dv = \log|f(v)|\right]$

$\therefore \log|\log v + 1| = \log|y| + \log c = \log|cy|$ $\therefore \log v + 1 = \pm cy = Cy$ where $C = \pm c$

Replacing v by $\frac{x}{y}$, we have

$$\log\frac{x}{y} + 1 = Cy$$

or $-\log\frac{y}{x} + 1 = Cy$ $\quad \left[\because \log\frac{x}{y} = -\log\frac{y}{x}\right.$ see page 632$\left.\right]$

Dividing by -1, $\log\frac{y}{x} - 1 = -Cy$ or $= C_1 y$ which is a primitive (solution) of the given D.E.

dy = 0

x/y

$^{x/y}$) dx + e

10. (1 + e

x/y

$^{x/y}$) dx + e

Sol. The given differential equation is (1 + e

+ e dy = 0

–

x/y

x/y

Dividing by dy, (1 + $^{x/y}$)

e

– = 0 –

$^{x/y}$)

or (1 + e +

= – e

– or= ∴ The given differential

...(i)

= f

which is a differential equation

equation is homogeneous.

of the form

Hence put = v i.e., x = vy Putting these values of

Differentiating w.r.t. y,

= v + y

in (i), we have

.

and

v + y

=

−

+ ·

Now transposing v to R.H.S.

y

v) dv = – (e

−

− − −v − −

=

+ – v = + =

+

v + v) dy or

∴ y (1 + e

dv = – +

+

$^{x/y}$ = C where C = ± c

x + y e

which is the required general solution.

^{v}) | = – log | y | + log | c |

Integrating, log | (v + e

Replacing v by, we have

+

log

or

= log

+ =

∴ + e$^{x/y}$ = ± !

Multiplying every term by y,

For each of the differential equations in Exercises from 11 to 15, find the particular solution satisfying the given condition: 11. (x + y) dy + (x – y) dx = 0; y = 1 when x = 1

Sol. The given differential equation is

(x + y) dy + (x – y) dx = 0, y = 1 when x = 1 ...(i) It looks to be a homogeneous differential equation because each coefficient of dx and dy is of same degree (here 1).

From (i), (x + y) dy = – (x – y) dx

–

+ =

=

∴ = +

+

– – –

or

− +

= f

=

  ...(ii)

∴ Given differential equation is homogeneous. Put

= v. Therefore y = vx.

∴

= v . 1 + x

= v + x

Putting these values in eqn. (ii), v + x

=−

+

−V = +

=

−− +

⇒ x − + =−− −

− −

+ = +

∴

⇒ x
= − +
+

+∴ x(v + 1) dv = −

$(v^2 + 1)$ dx

+ dv = −

Separating

variables,

+∫ dv +

⇒
+ ∫ dv = −∫ dx

⇒ log $(v^2 + 1)$ + $\tan^{-1}$
∵ ∫

+∫ dv + $\tan^{-1}$ v = − log x + c

Putting v = , log
+

v = − log x + c

′

+ $\tan^{-1}$ = − log x + c

⇒ log

+

  + $\tan^{-1}$ = – log x + c

$\Rightarrow$ [log ($x^2 + y^2$) – log x^2] + $\tan^{-1}$ = – log x + c $\Rightarrow$ log ($x^2 + y^2$) – 2 log x + $\tan^{-1}$ = – log x + c $\Rightarrow$ log ($x^2 + y^2$) + $\tan^{-1}$ = c ...(iii) To find c:
Given: y = 1 when x = 1.

Putting x = 1 and y = 1 in (iii), log 2 + $\tan^{-1}$ 1 = c π π = ⇒ =

   ∵

or c = log 2 + π

Putting this value of c in (iii),

log ($x^2 + y^2$) + $\tan^{-1}$ = log 2 + π

Multiplying by 2,

log ($x^2 + y^2$) + 2 $\tan^{-1}$ = log 2 + π

which is the required particular solution.

12. x^2 dy + (xy + y^2) dx = 0; y = 1 when x = 1

Sol. The given differential equation is

x^2 dy + (xy + y^2) dx = 0 or x^2 dy = – y (x + y) dx

 +

∴

+ = –

= –

= –

 +

  = F

  ...(i)

or

∴ The given differential equation is homogeneous. Put

= v, i.e., y = vx

Differentiating w.r.t. x,

= v + x

Putting these values of and

in differential equation (i),

we have v + x

R.H.S., x

= – v_2 – 2v

= – v(1 + v) = – v – v_2

Transposing v to

or x or

= – v (v + 2) x dv = – v (v + 2)

dx

v

+ = –

Integrating both sides,

+∫ dv = – ∫ dx

or

+∫ dv = – log | x | or

+ –

+∫ dv = – log | x |

Separating terms

or

  –

  +∫ dv = – 2 log | x |

or log | v | – log | v + 2 | = log x^{-2} + log | c |

+ = log | cx$_{-2}$|

or log

$=$

$+$ $\therefore$

$\therefore$

Replacing v to , we have

$+ = \pm$

$= \pm$ or

$+$

$+ = \pm$

or $x^2y = C(y + 2x)$

where C = ± c ...(ii) To find C

Put x = 1 and y = 1 (given) in eqn. (ii), 1 = 3 C ∴ C = Putting C = in eqn. (ii), required particular solution is

x^2

$y = (y + 2x)$ or $3x^2y = y + 2x$.

13.

dx + x dy = 0; y = π when x = 1

Sol. The given differential equation is

$-$ dx + x dy = 0; y = π, x = 1

$\Rightarrow$ x dy = $-$

Dividing by dx, x

$-$

dx

$= - x \sin_{2} + y$

Dividing by x,

$= -\sin^2 + \ldots$(i)

= F

□ □ □ □ □ ∴ The given differential equation is homogeneous.

Put = v ∴ y = vx ∴

= v . 1 + x

= v + x

Putting these values in differential equation (i), we have v + x

$= -\sin_2$ v + v ⇒ x

⇒ x dv = $-\sin^2$ v dx

Separating variables, $\int = -\int$ dx
$= -\sin_2$ v

= – Integrating,

⇒ – cot v = – log | x | + c

Dividing by – 1, cot v = log | x | – c

Putting v = , cot = log | x | – c ...(ii) To find c : y = π when x = 1 (given)

Putting x = 1 and y = π in (ii), cot π = log 1 – c or 1 = 0 – c or c = – 1

Putting c = – 1 in (ii), required particular solution is cot

= log | x | + 1 = log | x | + log e = log | ex |.

14.

– + cosec □ □ □ □ □

= 0; y = 0 when x = 1

Sol. The given differential equation is

– + cosec = 0; y = 0 when x = 1

 = – cosec = f

or ...(i) ∴ Given differential equation (i) is

homogeneous. Put = v ∴ y = vx ∴

= v . 1 + x

Putting these values in differential equation (i), =

–

v + x

= v – cosec v ⇒ x

∴ x sin v dv = – dx

Separating variables, sin v

Integrating both sides,

$\int$ = – $\int$ dx – cos v = – log | x | + c

dv = –

Dividing by – 1, cos v = log | x | – c

Putting v = , cos = log | x | – c ...(ii) To find c: Given: y = 0 when x = 1

∴ From (ii), cos 0 = log 1 – c or 1 = 0 – c = – c ∴ c = – 1

Putting c = – 1 in (ii), cos = log | x | + 1 = log | x | + log e ⇒ cos = log | ex | which is the required particular solution. 15. $2xy + y^2 - 2x^2$ = 0; y = 2 when x = 1

Sol. The given differential equation is

$2xy + y^2 - 2x^2$ = 0; y = 2 when x = 1. ...(i)

The given differential equation looks to be homogeneous because each coefficient of dx and dy is of same degree (2 here).

From (i), $-2x^2$

$= -2xy - y_2$ ∴

–

= –

= +

  ...(ii)

or

= F –

∴ The given differential equation is homogeneous. Put

= v ∴ y = vx ∴

= v . 1 + x

= v + x

Putting these values in differential equation (ii), we have v + x

$= v + v^2 \Rightarrow x$

$= v^2 \Rightarrow 2x\,dv = v^2\,dx$

=

Separating variables, 2

–

$\int$

$\int dx$

Integrating both sides, 2 –

dv =

Putting v = ,

⇒ 2

= log | x | + c

= log | x | + c ⇒

= log | x | + c

or = log | x | + c ...(iii)

To find c: Given: y = 2, when x = 1.

∴ From (iii), = log 1 + c or – 1 = c

Putting c = – 1 in (iii), the required particular solution is

= log | x | – 1

⇒ y (log | x | – 1) = – 2x ⇒ y =

⇒ y =

⇒ y =

.

16. Choose the correct answer:

A homogeneous differential equation of the form

= h

can be solved by making the substitution:

(A) y = vx (B) v = yx (C) x = vy (D) x = v Sol. We know that a homogeneous differential equation of the form = h

□ □ □ □ □ can be solved by the substitution = v i.e., x = vy.

∴ Option (C) is the correct answer.

17. Which of the following is a homogeneous differential equation?

(A) $(4x + 6y + 5)\,dy - (3y + 2x + 4)\,dx = 0$

(B) $(xy)\,dx - (x^3 + y^3)\,dy = 0$ (C) $(x^3 + 2y^2)\,dx + 2xy\,dy = 0$ (D) $y^2\,dx + (x^2 - xy - y^2)\,dy = 0$

Sol. Out of the four given options; option (D) is the only option in which all coefficients of dx and dy are of same degree (here 2). It may be noted that xy is a term of second degree.

Hence differential equation in option (D) is Homogeneous differential equation.

Exercise 9.6

In each of the following differential equations given in Exercises 1 to 4, find the general solution:

1.

+ 2y = sin x

Sol. The given differential equation is

+ 2y = sin x

| Standard form of linear differential equation

Comparing with

+ Py = Q, we have P = 2 and Q = sin x

∫ = e_{2x}

$ ∫ dx = ∫ dx = 2 ∫ dx = 2x I.F. = $ Solution is y(I.F.)

= ∫ + (I.F.) dx

dx + c

+ c or y e^{2x}

= ∫

or y e^{2x} = I + c ...(i) where I = ∫ Applying Product Rule of Integration

& && & && & &&

, dx ....(ii) I II

or I = – e^{2x} cos x + 2

$\int$ cos x dx

= e^{2x}(– cos x) – $-\int$ dx

I II

Again applying Product Rule,

I = – e^{2x} cos x + 2 – $\int$ ⇒ I = – e^{2x} cos x +

2e^{2x} sin x – 4 $\int$ or I = e^{2x}(– cos x + 2 sin x) – 4I

Transposing 5I = e^{2x}(2 sin x – cos x)

(2 sin x – cos x)

∴ I =

Putting this value of I in (i), the required solution is (2

sin x – cos x) + c

y e^{2x} =

Dividing every term by e^{2x}, y = (2 sin x – cos x) +

or y = (2 sin x – cos x) + c e^{-2x}

which is the required general solution.

2.

+ 3y = e^{-2x}

Sol. The given differential equation is

+ 3y = e^{-2x}

| Standard form of linear differential equation

Comparing with

$+ Py = Q$, we have $P = 3$ and $Q = e^{-2x}$

$\int = e_{3x}$

$\$\int$ dx = $\int$ dx = 3 $\int$ dx = 3x I.F. = $ Solution is y(I.F.)

$= \int +$ (I.F.) dx + c

or y $e^{3x} = -$

$\int$ x + c e^{3x} dx + c or = $- + \int$

dx + c or y e^{3x} = e dx + c = $\int$

Dividing every term by e^{3x}, y =

or $y = e^{-2x} + ce^{-3x}$

+
which is the required general solution.

3.

$+ = x^2$

Sol. The given differential equation is

$+ = x^2$

It is of the form

+ Py = Q Comparing P

$\$\int$ = , $Q = x^2$

dx = $\int$ dx = log x ∴ I.F. =$ $\int = e^{\log x} = x$

The general solution is y(I.F.) = $\int +$ (I.F.) dx + c

or yx = $\int$ dx + c = $\int$ (sec x) y =

dx + c or xy = + c. +

  π ≤ < tan x

4.

Sol. The given differential equation is

+ (sec x) y = tan x

It is of the form

+ Py = Q.

Comparing P = sec x, Q = tan x

$\int$ + tan x) = sec x

dx = $\int$ I.F. = + tan x

dx = log (sec x + $\int$ = e$^{\log (\sec x}$

tan x)

The general solution is y(I.F.) = $\int$ + (I.F.) dx + c

dx

or y (sec x + tan x) = $\int$ (sec x + tan x) dx + c = + $\int$

+ c = + − $\int$

dx + c = sec x + tan x − x + c

or y (sec x + tan x) = sec x + tan x − x + c.

For each of the following differential equations given in Exercises 5 to 8, find the general solution:

π ≤ <

+ y = tan x

5. $\cos^2$ x

Sol. The given differential equation is $\cos^2$ x

Dividing throughout by $\cos^2$ x to make the coefficient of unity,

+ y = tan x

+ ($\sec^2$ x) y = $\sec^2$ x tan x

⇒

+ =

It is of the form

+ Py = Q.

Comparing P = $\sec^2$ x, Q = $\sec^2$ x tan x

$\int$ dx = $\int$

dx = tan x I.F. = $\int$ = $e^{\tan x}$

The general solution is y(I.F.) = $\int$+ (I.F.) dx + c

or $ye^{\tan x} = \int \tan x \,.\, e^{\tan x}$ dx + c ...(i) Put tan x = t. Differentiating $\sec^2$ x dx = dt

I I I

$\therefore \int \tan x\, e^{\tan x}$ dx = $\int$

e^t dt

Applying integration by Product Rule,

t – $\int$. e

t dt = t . e

t – e

t = (t – 1) e

= t . e

t = (tan x – 1) $e^{\tan x}$ Putting

this value in eqn. (i), $ye^{\tan x}$ = (tan x – 1) $e^{\tan x}$ + c Dividing every term by $e^{\tan x}$,

y = (tan x – 1) + $ce^{-\tan x}$ which is the required general solution. 6. x

+ 2y = x^2 log x

Sol. The given differential equation is x

+ 2y = x^2 log x

Dividing every term by x

,- #

+ y = x log x

It is of the form log x $ \int

+ Py = Q.

Comparing P = , Q = x dx = 2 $\int$ dx = 2 log x$\int$ = e 2

$e^{\log f(x)}$ = f (x) log x = $e^{\log x^2}$ = x^2 ∴

I.F. = $

The general solution is y(I.F.) = $\int$+ (I.F.) dx + c

or $yx^2 = \int . x^2$ dx + c = $\int . x^3$ dx + c

$\int$ dx + c

= log x .

$- \int$. dx + c = log x – log x –

(

or yx^2 = log x –

Dividing by + (

x^2, y =

+ c. y = . (

(4 log x – 1) +

+ y = log x

7. x log x

Sol. The given differential equation + y = log x
is x log x

Dividing every term by x log x to make the coefficient of

+

unity,

Comparing with + Py = Q, we have

P = and Q =

y =

$\int$ dx =

$\int$ dx =

$\int$ dx = log (log x)

$\therefore \int$. $\int$ dx + c or y log x = $\int$ log x

I.F. = $\int$ = e$^{\log (\log x)}$ = log x The general solution is y(I.F.) = + dx = 2 −

dx + c I II

$\int$

Applying Product Rule of integration,

− −

= 2 − − − + c = 2

= 2

− = (1 + log x) + c.

− + − + c or y log x

− − + + c

8. $(1 + x^2)$ dy + 2xy dx = cot x dx $(x \neq 0)$

Sol. The given differential equation is $(1 + x^2)$ dy + 2xy dx = cot x

dx Dividing every term by dx, $(1 + x^2)$

+ 2xy = cot x

Dividing every term by $(1 + x^2)$ to make coefficient of unity,

+

+ y =

+

Comparing with

+ Py = Q, we have

P =

+ and Q =

+

′

+ $\int$ dx = log | 1 + x^2|

$ $\int$ dx = ∵ $\int$

= log (1 + x^2) [∵ 1 + x^2 > 0 ⇒ | 1 + x^2| = 1 + x^2] $\int$ = $e^{\log(1+x^2)}$ = 1 + x^2

I.F. = $

dx + c

Solution is y(I.F.)

= + & . $\int$

+ $\int$ (1 + x^2) dx + c

⇒ y(1 + x^2) =

⇒ y(1 + x^2) = $\int$ + c ⇒ y(1 + x^2) = log | sin x | + c

+ + +

Dividing by 1 + x^2, y =

or y = $(1 + x^2)^{-1}$ log | sin x | + c $(1 + x^2)^{-1}$

which is the required general solution.

For each of the differential equations in Exercises 9 to 12, find the general solution:

= 0, (x ≠ 0)

9. x

+ y – x + xy cot x

Sol. The given differential equation is

x

+ y + xy cot

+ y – x + xy x = x

cot x = 0

⇒ x ⇒ x + (1 + x cot

x) y = x

Dividing every term by x to make coefficient of

unity,

+

+

y = 1

Comparing with

+ Py = Q, we have

+ and Q = 1

P =

+ $\int$

dx =

□ + □ □

□ □ $\int$ dx =

$ = $\int$

□ + □ □

□ □ $\int$ dx

⇒ $ $\int$ = log x + log sin x = log (x sin x)

I.F. = $

$\int$ = e log (x sin x) = x sin x

+ c

Solution is

y(I.F.) = + & . $\int$

or y(x sin x) = $\int$ "" # $ % * & && & && & &&

dx + c

I II

□ = − □ □□ □

□ □ □ □ □ $\int$ $\int\int\int$ ⇒ y(x sin x) = x(– cos x) – −

$\int$ dx + c

= – x cos x + $\int$ + c

or y(x sin x) = – x cos x + sin x + c

which is the required general

Dividing by x sin x, y =
+

10. (x + y)

= 1

− +

Sol. The given differential equation is

(x + y)

or y = – cot x + +
solution.

= 1 ⇒ dx = (x + y) dy

⇒ = x + y ⇒

– x = y

| Standard form of linear differential equation

Comparing with

+ Px = Q, we have, P = – 1 and Q = y

$ ∫ = − ∫ = − ∫ = – y I.F. = $ ∫ = e

– y ∴ Solution is x(I.F.) = +

& . ∫ + c

or xe^{-y} = – dy + c I II

∫

= − ∫ ∫∫∫ "" # $ % * & && & && &