Exercise 7.1

Find an antiderivative (or integral) of the following functions by the method of inspection in Exercises 1 to 5. 1. sin 2x

Sol. To find an anti derivative of sin 2x by Inspection Method.

We know that $\frac{d}{dx}(\cos 2x) = -2 \sin 2x$

Dividing by – 2, $-\frac{1}{2}\frac{d}{dx}(\cos 2x) = \sin 2x$

or $\frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right) = \sin 2x$

∴ By definition; an integral or an antiderivative of sin 2x is $-\frac{1}{2}\cos 2x$.

Note. In fact anti derivative or integral of sin 2x is $-\frac{1}{2}\cos 2x + c$. For different values of c, we get different antiderivatives. So we omitted c for writing an anti derivative.

2. cos 3x

Sol. To find an anti derivative of cos 3x by Inspection Method. We

d

know that cos 3x
dx (sin 3x) = 3

Dividing by 3, dx (sin 3x) = cos 3x or dx 1 sin 3 3 x
1 3
d d
□ □ □ □ □ = cos 3x

∴ By definition, an integral or an antiderivative of cos 3x is $\frac{1}{3}$sin 3x.

(See note after solution of Q.No.1 for not adding c to the answer.) 3. e^{2x}.

Sol. To find an antiderivative of e^{2x} by Inspection Method. We

know that d dx $e^{2x} = e^{2x}$ d dx (2x) = $2e_{2x}$

d d
Dividing by 2, $\frac{1}{2}$ dx e^{2x} = e^{2x} or e dx $\frac{1}{2x}$ □ □ □ □ □= e^{2x} 2

∴ An antiderivative of e^{2x} is $\frac{1}{2}$ 4. (ax + b)^2. e^{2x}.

Sol. To find an anti derivative of $(ax + b)^2$.

We know that d dx $(ax + b)_3$ = $3(ax + b)^2$ d dx (ax + b) = 3(ax + $b)_2$a.

d
Dividing by 3a, $\frac{1}{3a}$ dx $(ax + b)_3$ = (ax + b)^2
1 dx 3 ( )3

d ax b a
or
□ □ + □ □ □ □= $(ax + b)^2$

∴ An anti derivative of $(ax + b)^2$ is $\frac{1}{3a}(ax + b)_3$.

5. sin 2x – $4e^{3x}$
.

Sol. To find an anti derivative of sin 2x – $4e^{3x}$ by Inspection Method. . 2

d
We know that Dividing by – 2,
2 x □ –
d
dx (cos 2x) = – 2 sin 2x dx 1 cos 2

= sin 2x ...(i)

Again d dx e^{3x} = $3e^{3x}$ ∴ d dx $\frac{1}{3}$ e^{3x}

Multiplying by – 4,

d dx $-\frac{4}{3} e^{3x}$ = – $4e^{3x}$ ...(ii)

Adding eqns. (i) and (ii)

d dx $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right)$ = sin 2x – $4e^{3x}$

or d dx $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right)$ = sin 2x – $4e^{3x}$

∴ An anti derivative of sin 2x – $4e^{3x}$ is $-\frac{1}{2}$ cos 2x – $\frac{4}{3}$ e^{3x}.

Evaluate the following integrals in Exercises 6 to 11.

6. $\int (4e^{3x} + 1)\,dx$.

Sol. $\int (4e^{3x} + 1)\,dx = 4\int e^{3x}\,dx + 1\int dx$

$= 4\frac{e^{3x}}{3} + x + c$. $\left[\because \int e^{ax}\,dx = \frac{e^{ax}}{a} \text{ and } \int 1\,dx = x\right]$

7. $\int x^2\left(1 - \frac{1}{x^2}\right)dx$

= 4 3x e ∫ dx.

 − ∫ dx = 2

 − ∫ dx = 2(1) x − ∫ dx

Sol. 2 2 1 x 1 x x x 2 x 2 n n x x dx n n+

dx − 1 ∫ x − x + c. 1 if

= 2 x ∫ dx = 3 3 1

∵ ∫

= ≠ −

+

1

8. ∫ 2(+ +) ax bx c dx .

Sol. 2( ) ax bx c + + ∫ dx = 2 ax ∫ dx + bx ∫ dx + c ∫ dx = a 2 x ∫ dx + b 1 x where c_1 is the constant of

∫ dx + c 1 ∫ dx = a 3 3 x + b 2 2 x + cx + c1

integration.

9. 2 ∫ 2(+) x x e dx.

Sol. 2(2 + x x e ∫ x dx ∫ + dx)

dx = 2 2 x e ∫ x+

= 2 2 x ∫ dx + x e ∫ dx = 2 2 1 10. 3

x + c = 2 x + e x + c. 3

+ + e
2 1

∫

$\int \frac{(1-x)^2}{x}\,dx$. Sol.

. 3

2

1

x

x

$- \int dx$

Opening the square =

2

$+ - \int dx$

1 1 () 2 x x

x x +dx = x $\int$ dx

x $\int$ dx

2

– $\int$ dx + 1 – 2 $\int$

= 1

x^2

x

2

= x + log | x | – 2x + c. 2 21 2 dx dx

x = = ∵ $\int \int$ 2

11. $\int \frac{3x^2}{x^2 + 5x - 4}$

dx.

x

2

3 2

dx =3 2 + − ∫

  ∫

 + − dx

x x5 4

Sol.

x x5 4 xxx

x 2 222

 abc a b c

+ −=+−

  Using d ddd

x − + − ∫ dx + 5∫ x− +

dx

$=^2$(54) x dx = 1 x∫ dx − 2 4x− ∫

dx = 2 2x + 5x − 42 1

∫

2

2

= = x + 5 1∫

x+ 5x +4 − + + c 2 1

+ c.

x

dx − 4 2 x−2

2

Evaluate the following integrals in Exercises 12 to 16. 12. ∫3 x x

+3 +4

dx.

x

3 x x3 4

dx + +

 ∫

Sol. x x3 4 xxx

x

+ +∫ dx =3 1/2 1/2 1/2

=3 1/2 1 1/2 1/2(3 4) xxx − −− + +∫ 4) xxx − + +∫

dx = 5/2 1/2 1/2 (3 dx=

5/2 x∫ x∫ x− dx

dx + 3 1/2 dx + 4 1/2

∫

5/2 1

x+ 1/2 1

1/2 1 x + 3 3/2

= + 3 x− + + 2 1 1 2 7 2

x + 1 1 + 4 − + c = x + 4 1/2 x + c 2 3 2 1 2

5 1 + 7/2 +

$= \frac{2}{7} x^{7/2} + 2x^{3/2} + 8x^{1/2} + c.$

13. ∫ 3 2 − + −1

xxx

x x

.

3 2 1 −

d 1

2 (1)(1)

− ∫ dx = 2 (1) (1) x x

− +− xxx − +

−+ −

x

xx x 1

x ∫ dx 1 Sol.

− ∫ dx = 2 (1)

x + ∫

dx

=

(1)

x

x+

= 2 x ∫ dx + 1 ∫ dx = 2 1 Sol. (1) − 3 x x ∫ −)

∫(dx dx = x xx

2 1

14. ∫(1 −) x x dx.

+ + x + c = x + x + c. 3

=1/2 1 1/2 x xx –)∫(dx = 1/2 1 1/2 x x) + −∫(dx

=1/2 3/2 x x − 1 1 1 x + 3 1

3/2 1

)∫(dx = 1/2 x_5/2 – + +

x+ 2

+ c 2

3/2 3 2 x^5/2 + c.

5 2

2

= x+ c = 3

x^3/2 – 2 5

15. $\int x^2 (3x + 2x + 3)\,dx$.

Sol. 2 xx x (3 2 3) + + ∫ + ∫

dx = 1/2 2 xxx (3 2 3) + dx= 2 1/2 1/2 1/2 (3 2 3) x x

xx x + + ∫ 1/2 (3 2 3) xxx + dx + + += = += 21 3 2 , 1

dx = 5/2 3/2 + ∫ 222 222

= ∵ 1 41 5 1

= 3 5/2 x ∫ dx + 2 3/2 x ∫ dx + 3 1/2 x ∫ dx

3/2 1 1/2 1

5/2 1 x + 2 5/2

x+ x+ x+ 7/2

x + 3 3/2 2

= 3 +

2 x+ c 3

5 1 + + 2 3 1 + + 3 1 1 2 7 2 5 2 2

+ c = 3

$= \frac{6}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2} + c.$

16. ∫(2 – 3 cos x + e^x) x dx.

Sol. (2 3 cos) x x xe − + ∫ dx = 2x ∫ dx – 3 cos x ∫ dx + e x ∫ dx = 2 1 x ∫

dx – 3 cos x ∫ dx + $_x$ e ∫ x– 3 sin x + e

x + c = x^2 – 3 sin x + e dx = 2^2 2

x

+ c.

Evaluate the following integrals in Exercises 17 to 20. 17. $\int^2$(2 – 3 sin + 5) x xx dx.

Sol.^{2}(2 3 sin 5)dx

x xx − + ∫

x^+ x ∫ x ∫

= 2^2 x ∫ dx – 3 sin dx + 5 $_{1/2}$ dx

cosx) + 5 3 3

1/2 1 x+ 3 cos x +2 1 3 3 2

= 2

2 1 = 2

x + 1 1 + c= $5^{3/2}$ x+ c 2

+– 3(– 2 3 +

18. ∫sec (sec + tan) xx x dx.

. 5

x+ 3 cos x + $\frac{10}{3}$ $x^{3/2}$ + c.

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tan) xx x + ∫ tan) x xx + ∫ dx

dx = 2 (sec sec dx= ^{2}sec

x ∫

Sol. sec (sec

19. $\int^2$ 2

+ sec tan x x ∫

dx = tan x + sec x + c.

sec x cosec x dx.

$\sec^2$ x 1_2 dx = 2 x sin

x

x ∫

Sol. dx ∫ 2

cos cos dx = 1 $\sin^2$ x

x ∫ cosec 2

=^{2}tan dx = $\int^2$(sec

x ∫ – 1) x dx

x ∫ (∵ $\sec^2 x - \tan^2 x$ $\tan^2 x)$ dx = tan x

$= ^2\sec$ dx – 1 ∫ = 1 ⇒ $\sec^2 x - 1$ = – x + c.

dx = ^{2}(cosec dx = – cot x

Note. 1) x $^-$ ∫ 20. ∫ 2 cos – x + c. 2 –

Similarly^2 dx x ∫ 3 sin x

cot

x ∫ = ^{2} cosec dx – 1 ∫ x

x dx.

x

2 3 sin

– ∫ dx = 2 2 – ∫ dx

2 3 sin cos cos

Sol. 2

cos x x x = 2 3 sin 2

sec –

cos cos x dx = 2 (2 sec 3 tan sec dx

∫

x) x xx – ∫

x x

x ∫ x x ∫ sec x + c. 21.

dx – 3 sec tan dx = 2 tan x – 3

= 2 2^2sec

Choose the correct answer:

1

The anti derivative of

x +

x equals

(A) $\frac{1}{3} x^{1/3} + 2x^{1/2} + C$ (B) $\frac{2}{3} x^{2/3} + \frac{1}{2} x^2 + C$ (C) $\frac{2}{3} x^{3/2} + 2x^{1/2} + C$ (D) $\frac{3}{2} x^{3/2} + \frac{1}{2} x^{1/2} + C$.

Sol. The anti derivative of the 1 +

x

x

= 1

+ ∫ dx = 1/2

x

x

∫

1/2 ( ) x x − + ∫ dx = 1/2 1 1/2 1

dx

dx + 1/2 x + 1 1 2 + 2

x −

= 1/2 x ∫ x + 1 1 + + C x + 1/2

+

$x^{3/2} + 2x^{1/2}$

2 + C

= 2

x + C = 3 1 2

3/2 3

∴ Option (C) is the correct answer.

22. Choose the correct answer:

If $\frac{d}{dx}$ f (x) = $4x^3 - \frac{3}{x^4}$ such that f (2) = 0. Then f (x) is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$

(B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$

(D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

Sol. Given: $\frac{d}{dx}$ f (x) = $4x^3 - \frac{3}{x^4}$

and f (2) = 0 ∴ By definition of anti derivative (i.e., Integral),

f (x) = $\int \left(4x^3 - \frac{3}{x^4}\right) dx = 4\int x^3 dx - 3\int x^{-4} dx$

x^{-}

$dx = x^{4} - 3^{3}3$

$\int$

= 4 .

$x _{-3}^{4} x^{-} 4$ + c

or $f(x) = x^4 + {}_3{}^1 (\ ) x^{+c}$ ...(i) To find c. Let us make use of f (2) = 0 (given)

Putting x = 2 on both sides of (i),

$f(2) = 16 + {}^1 8 + c$ or $0 = 128\ 1$

8

+ + c

f (2) = 0 (given)) ($\because$

–

or c + 129

8 = 0 or c = 129

8

– in (i), $f(x) = x^4 + {}_3{}^1 (\ )\ x$ – 129

Putting c = 129

8

8

$\therefore$ Option (A) is the correct answer.

Exercise 7.2

Integrate the functions in Exercises 1 to 8: 2 x

1. 2

1 + x 2 x

dx + x ∫

Sol. To evaluate 2

1 dt

Put $1 + x^2 = t$. Therefore dx or 2x dx = dt

2x =

2 x

dt

t∫ = 1 t∫ dt = log | t | + c

∴ 2

+ x ∫ dx =

1

Putting t = 1 + x^2, = log | 1 + x^2| + c = log (1 + x^2) + c. (∵ 1 + x^2 > 0.
Therefore | 1 + x^2| = 1 + x^2)

^{2}(log) x

2.

x·

^{2}(log) x

Sol. To evaluate

x ∫ . 8

dx

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Put log

dx

x = t. Therefore 1

dt

x

x =

dx ⇒

= dt

^{2}(log) x

dt = ^{3} 3

∴

t ∫

t + c

dx = 2

x ∫

Putting t = log x, = $\frac{1}{3}$(log x)^3 + c. 3. 1

xx x + log

dx =

1

x x (1 log) + ∫ dx

xx x +

log ∫

Sol. To evaluate 1

= t.

∴ 1

Therefore 1

dx

dt

x =

Put 1 + log x

dx

+ c

x = dt

dx ⇒

x =1

1 log + x∫ t∫ dt = log | t |

xx x + log ∫ dx =1

Putting t = 1 + log x, log | 1 + log x | +

c. 4. sin x sin (cos x)

Sol. To evaluate sin sin (cos) x x∫ dx = – sin (cos) (sin) x x −∫ dx

Put cos x = t. Therefore – dtdx∴ – sin x dx = dt
sin x =

∴ sin sin (cos) x x∫ dx = – sin (cos)(sin) x x dx −∫ = – sin t∫ dt = – (– cos t) + c
= cos t + c
Putting t = cos x,
= cos(cos x) + c.

5. sin (ax + b) cos(ax + b)

cos( ) ax b ax b + +∫ dx

Sol. To evaluate sin( )

$= \frac{1}{2}$ 2 sin( ) cos( ) ax b ax b + +∫ dx = $\frac{1}{2}$ sin 2( ) ax b +∫ dx (∵ 2 sin θ cos θ = sin 2θ)

$= \frac{1}{2}$ sin (2 2) ax b +∫ = 1 dx = $\frac{1}{2}$[cos(2 2)] ax b − +

→ + c

2 Coeff. of a x

− cos 2(ax + b) + c.
4a

6. ax + b

Sol. To evaluate ax b +∫ dx = 1/2 ( ) ax b +∫ dx

→

32( )

2( ) +

ax b

+ c

ax b

+

=

=

+

1

1 Coeff. of

+ c

a

3

2

ax^2

n

ax b ax b

dx c n

+ = +

≠−

a n

+

n

∵ ∫ 1 ( )

( ) if 1

+

(1)

$= \frac{2}{3a}(ax + b)^{3/2} + c.$

7. x x + 2

Sol. To evaluate $\int x\sqrt{x+2}\,dx$

$= \int x\sqrt{x+2}\,dx = \int ((x+2)-2)\sqrt{x+2}\,dx$

dx = 3 1

= =

+ +−+ 32(2) x + ∫

2 2(2) 2(2) x x

dx

dx +−+

∫

1 1

∫

dx

2 2(2)(2) 2(2) xx x 3 1

+

2(2) 1 1

x + 2(2) 52(2) 32(2)

dx – 2 12 (2) x + ∫

= x + +

x + – 2 x + + c

+ → – 2 +

c = 5

3

3

1 1 Coeff. of 2 x

11 .1 2 2 2

$= \frac{2}{5}(x + 2)^{5/2} - \frac{4}{3}(x + 2)^{3/2}$ + c. OR

To evaluate dx

x x + 2 ∫

Put Linear = t, i.e., x + 2 = t.
Squaring x + 2 = t^2(⇒ x = t^2 – 2)

∴ dx dt = 2t, i.e., dx = 2t or dt

dx = 2t

dt ∴ x x + 2 ∫ dx = 2 (2) . 2 t tt − ∫ dt = 2 2 2 (2)

t t − ∫ dt t ∫ dt = 2 5 5 t – 4 3 3

= 2 2 2 (2) t t − ∫ dt = 2 4 t + c t ∫ dt – 4 2

Putting t = x + 2 , = 2 5 5(2) x + – 4 3 3(2) x + + c = 2 5(x + 2)^{1/2})^5 – 4 3((x + 2)^{1/2})^3 + c = $\frac{2}{5}(x + 2)^{5/2} - \frac{4}{3}(x + 2)^{3/2}$ + c. 8. x^2 1 2 + x

Sol. To evaluate 2 x x 1 2 + ∫ dx

Let I = 2 x x 1 2 + ∫ x x dx ∫ ...(i)

dx = 1 4 2 1 2 (4) + 2 4 2(1 2) 0 2 .

□ □ + =+ = □ □ □ □ ∵

dt

Put 1 + $2x^2$= t. Therefore 4x = d x xx dx or 4x dx = dt

dt = 1 4 1/2 t ∫

t ∫ dt

∴ From (i), I = 1 4 . 10 = 1 4 3/2

t+ c = 1 4 . 2 3 $t^{3/2}$ + c

3

2

Putting t = 1 + $2x^2$, = 1 6$(1 + 2x^2)^{3/2}$ + c.

Integrate the functions in Exercises 9 to 17: 9. $(4x + 2)^2$ x x + +1 .

Sol. Let I = 2(4 2) 1 x xx + ++∫ dx = 2 2(2 1) 1 x xx + ++∫ dx = 2 2 1 x x + +∫(2x + 1) dx ...(i) Put x^2 + x + 1 = dtdx ∴ (2x t. Therefore (2x + 1) = + 1) dx = dt

∴ From (i), I = 2 t∫ dt = 2 t∫ 1/2 dt 3/2 3 2

= 2

t+ c = 4 3 $t^{3/2}$ + c

Putting t = x^2 + x + 1, I = 4 3$(x^2 + x + 1)^{3/2}$ + c.

10. 1 x x –

Sol. Let I = 1 x x –∫ dx ...(i) Put Linear = t, i.e., x = t

Squaring x = t^2. Therefore dx dt = 2t or dx = 2t dt

∴ From (i), I = 2 1 t t t –∫ dt

t t – ∫2t dt = 2 (1)

= 2 1 t – 1 ∫ dt = 2 log | t – 1 | + c 1 1 dx ax b log| | ax b a

11. 4

x, I = 2 log | x − 1 | + c.

Putting t = x

x +, x > 0

x

Sol. Let I = 4 x + ∫ dx ...(i)

x + + ∫ dx

+ ∫ dx = 4 4

x

= 4 4

+ − 4 4

x x

4

x

= x + 4 ∫ 1

x + 4 ∫ dx t tt tt t dx − 4

Integrals dx

dx − 4 − + ∫

∵ t tt t

. 11

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= + ∫ + _

= + + c = (x + 4)^{3/2} − 8(x + 4)^{1/2} + c

= (x + 4) + − 8 + + c

+ − + c

+ − + c = 2 +

= 2 +

= + (x – 8) + c.

OR

Put = t, i.e., + = t.

Squaring $x + 4 = t^2 \Rightarrow x = t^2 - 4$.

Therefore = 2t or dx = 2t dt

∴ I = + ∫ dx =

− ∫ . 2t dt

= 2 − ∫ dt = 2 − ∫ ∫

− + c = $(t^2 - 12) + c$. =

2

Putting t = + , = + (x + 4 – 12) + c = + (x – 8) + c.

12. $(x^3 - 1)^{1/3} x^5$

Sol. Let I = − ∫ x^5 dx = − ∫ x^3 x^2 dx = − ∫ $(3x^2\, dx)$ ...(i)

Put $x^3 - 1 = t \Rightarrow x^3 = t + 1$ ∴ $3x^2$ =

$\Rightarrow 3x^2\, dx = dt$ ∴ From (i), I = + ∫ dt

− = ∵ +

dt

= + ∫

+= = ∵

= ( ) + ∫ ∫

$= \frac{1}{3}$ 7/3 4/3

t t + □ □ 7 $t^{4/3}$ + c

4

1 4

□

□ □ □ □ 3 3

Putting t = x^3 – 1, = $\frac{1}{7}$(x^3 – 1)$^{7/3}$ + $\frac{1}{4}$(x^3 – 1)$^{4/3}$ + c.

□ +

□ □ □ □ + c = $\frac{1}{7}$ t

x^2

Sol. Let I =

t

+ c = $\frac{1}{3}$ 3 3 7/3 4/3

7 4

$t^{7/3}$ +

13.

3 3(2 + 3 + x∫ dx 3

)

x 2

x

$= \frac{1}{9}$ 2 3 3

^{3}(2 3)

dx

9

□ □ + = □ □ □ + x∫ 3) 9 x

d x x

□ ∵ dx ...(i) 3 2 (2

(2 3)

$9x^2$ =

dt

dx ⇒ $9x^2$ dx = dt t

Put 2 + $3x^3$ = t. Therefore

∫ $\frac{1}{9}$ 2 2

dt =

∴ From (i), I + c – + c = $_2$1 18t

$= \frac{1}{9}$ 3 1 t –

Putting t = 2 + $3x^3$; = 3 2

+ + c.

14.[1]

m

x x', x > 0 (Important)

1

dx

Sol. Let I =1

x x ∫ dx (x > 0) ⇒ I =

x

m

(log)

(log)

Therefore1 x∫...(i)

dt

m

(log)

dx

Put log x = t.

t

dt

x = x = dt

dx ⇒ dt−

m

∴ From m

(i), I = m t

− +

1

t∫ =

m

x

∫= − + + c

1(log)−

m

1

(Assuming m ≠ 1)

1

x

Putting t = log x, = − + c. m

15.[2]9–4

x

−

Sol. Let I = 2 9 4x

x

− x ∫ dx = 1 8 − 2

− ∫ dx ...(i)

9 4 x d x x − =− ∵

dx

^{2}(9 4) 8 dt

Put $9 - 4x^2 = t$. Therefore $-8x = \frac{dt}{dx} \Rightarrow -8x\,dx = dt$ ∴ From (i), I $= -\frac{1}{8}\int \frac{dt}{t} = -\frac{1}{8}\log|t| + c$ Putting $t = 9 - 4x^2$, $= -\frac{1}{8}\log|9 - 4x^2| + c.$

16. e^{2x+3}

Sol. $\int e^{2x+3}\,dx = \frac{e^{2x+3}}{2} + c$ $\left[\because \int e^{ax+b}\,dx = \frac{e^{ax+b}}{a} + c \text{ and } a = 2 = \text{Coeff. of } x\right]$

$= \frac{1}{2}e^{2x+3} + c.$

17. $\frac{x}{e^{x^2}}$

e ∫ dx = $^1 2^2$

Sol. Let I...(i) ( )
= ^{2}( ) x
x

e ∫ dx

Put x^2= t. Therefore 2x = $^1 2$

= ∴ From (i), I = ^{1}2 ( )
t
dt
t e$^-$

dx ⇒ 2x dx = dt. dt e ∫

dt –
∫
t e

– → + c =1

= ^{1}2 1 Coeff. of

t
–
+ c. 1

Integrate the functions in

18.

Putting t = x^2, I =

26: tan
2 1 +
$_2$ ^{x} e
–x ^{x} e
2( )

Sol. Let I = ^{1}tan
–
+∫ x

t e

dx ...(i) 2 1

– + c 2( )

x

Exercises 18 to

= t.

⇒
2 1 dx 1

x e

Put $\tan^{-1}$ x dx
1 +
x =

∴ 2
dt

+ x = dt

2 2 x x

e

∴ From (i), 2 dt = e t + c 1x + c. –

19. I = t e ∫ = e tan – 1

e e

– 1 +1

x

Sol. Let I = +∫ dx e^{2x} 1

Multiplying every term in integrand by e^{-x}, x x

e e – –

x] x x

I = = e

+∫ dx ...(i) [∵ $e^{2x} . e^{-x} = e^{2x-x}$

e e –

x + e^{-x} = t

Put denominator e

x + e – x d ∴ e dx (– x) = – x) dx = dt ∴

dt ∫ =1 dt = log | t | + c

x – e dx ⇒ (e From (i), I = t

dt t∫

x + e^{-x}) + c 1 0 for

x + e^{-x}, I = log | e

Putting t = e all real and hence| |

x xx x xx x x ee e x ee ee – – –

x + e^{-x}| + c or I = log (e

+ =+ > + =+
  ∵ e

( )

2 –2

x x

–

e e

20. 2 –2

x x

e e

+ x x 2 2 – 2( ) x x

$+\int$ $\frac{1}{2}$ $2^{2\,2}$

dx =

x x 2 2 – $+\int$ e^{2x} d

Sol. Let I =

2 2

e e – e e – ⁻ dx ...(i) x x dt

e e e 2x d

Put – dx (– 2x) =

denominator e e -

2x + dx

dx $e^{2x} + e^{-2x} = t$

∴

dt

dx ⇒ $2(e_{2x} - e^{-2x})$ dx = dt

∴ From

⇒ $e^{2x}. 2 - 2e^{-2x}$ =

$t\int = \frac{1}{2}\log | t | + c$

dt

(i), I = $\frac{1}{2}$

Putting $t = e^{2x} + e^{-2x}$, = $\frac{1}{2}\log| e^{2x} + e^{-2x}| + c = \frac{1}{2}\log(e^{2x} + e^{-2x}) + c$

21. $\tan^2(2x - 3)$ [∵ $e^{2x} + e^{-2x} > 0$ ⇒ $| e^{2x} + e^{-2x}| = e^{2x} + e^{-2x}$]

Sol. $\int \tan^2 (2x - 3)$ dx (∵ $\tan^2\theta = \sec^2\theta - 1$) =

dx = $\int (\sec^2 (2x - 3) - 1)$

$\int \sec^2 (2x - 3)$ dx – $1\int$ dx = tan (2x – 3)

$- x + c = \frac{1}{2}\tan (2x - 3) - x + c$

→

–

2 Coeff. of

$\int \sec^2 (ax + b)\, dx = \frac{1}{a}\tan (ax + b) + c$

22. $\sec^2(7 - 4x)$ x

of

Sol. ^{2}sec (7 4) – ∵

x dx ∫ = tan (7 4) ∫ x

– → + c

–

x

4 Coeff.

1 sec ( ) ax b dx a

ax b c

2 tan ( )

∵ ∫

$= \frac{1}{4}$ – tan (7 – 4x) + c.

$\sin^{-1} x$

23. 1 – $_2$sin$_-$ x

x $_1$

Sol. Let I = 1

– ∫ dx ...(i) ⇒

1 dx 2

x dt

$_2$1 – x =

Put $\sin^{-1}$ x = t ∴ $_2$

dx x = dt 1

(i), I =

t ∫ t + c

dt = $\frac{2}{2}$

∴ From

Putting t = $\sin^{-1}$

24. 2 cos – 3 sin

x x

6 cos + 4 sin

x x

. 15

x, I = $\frac{1}{2}(\sin^{-1} x)^2$ + c.

Sol. Let I = 2 cos 3 sin

+∫ dx = 2 cos 3 sin

x x

−

−

x x

6 cos 4 sin

−

x x

+∫ dx 2(2 sin 3 cos)

= $\frac{1}{2}$ 2 cos 3 sin

x x

x x

sin 3 cos x

x

+∫

dx ...(i) 2

Put DENOMINATOR 2 sin x + 3 cos x = t

dt

∴ 2 cos x – 3 sin x =

dt ∴ From

dx ⇒ (2 cos x – 3 sin x) dx =

t∫ = $\frac{1}{2}$ log | t | + c.

dt

(i), I = $\frac{1}{2}$

Putting t = 2 sin x + 3 cos x, = $\frac{1}{2}$ log | 2 sin x + 3 cos x | + c. 1

25. 2 2

cos (1 – tan) x x

cos (1 tan) x dx = $\frac{2}{2}$

x − ∫

1 sec x

Sol. Let I = 2 2

−

x∫

dx (1 tan)

2

x

= −

− sec

x

(1 tan)

− ∫

dx ...(i)

Put 1 – $\tan^2$ x = t. ∴ − dt

$\sec^2$

x =

dx ⇒ − $\sec_2$ x dx = dt

dt

t⁻

∴ From (i), I = $t\int = -^2$

− 2

$\int$ dt

t⁻

1

= −

− + c =

1

t + c = 1

1 tan − x + c.

1

26. cos x

x

Sol. Let I = cos x

x $\int$ dx ...(i) Put Linear = t, i.e., x = t

Squaring, $x = t^2$. Therefore cos t =

dx

dt = 2t ∴ dx = 2t dt ∴ From (i), I

dt = 2 sin t + c x, I = 2 sin x + c. $t\int$

Putting t = $t\int$ 2t dt = 2 cos

Integrate the functions in Exercises 27 to 37: 27. sin 2x cos 2x

Sol. Let I = sin 2 d x $\int$ cos 2x dx = $\frac{1}{2}$ sin 2 dt

x $\int$ (2 cos 2x dx) ...(i) Put sin 2x = t

∴ cos 2x

dx (2x) =

dx ⇒ 2 cos 2x dx = dt . 16

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∴ From (i), I = $\frac{1}{2}$ dt = 1 $2^{1/2}$ $t\int$ dt

1 1

$t\int$

28. cos

3

= $\frac{1}{2}$

t^+

2

2

x

1 1

t + c = 1 3 (sin 2x)$^{3/2}$ + c.

+ x ∫ dx ...(i) 1 sin

1 + sin x

+

2

x

cos

+ c = 1 2 3/2

Sol. Let I =

sin x = t t⁻ dt t

Put 1 + ∫ = 1/2

∴ cos x = dx or cos x = dt t

dx = dt ∴ 1 1

From (i), I

dt =

∫

− + + c 1 1

1/2

2

c. 1

= − +2

2

29. cot x log sin x

+ c = 2

t + c = 2 1 sin + t

x +

log sin ∴1 sin x

dx ...(i) Put log

Sol. Let I = cot

sin x = t dt

x x ∫

d

dx or 1

dx

dx (sin x) =

dt

sin x cos x =

∴ From (i), I =

t + c = 1 2 (log sin x)^2 + c.

or cot x dx = dt

t ∫

dt = 2 2

1 + cos

x

x

Sol. Let I = sin

30. sin

dx = –

+ x ∫ x − x

sin

1 cos

+ ∫ x

dx ...(i)

1 cos

Put 1 + cos x = t. Therefore – sin x = dt

dtdx ∴ – sin x dx = dt

x, = – log | 1 + cos x | + c.

∴ From (i), I = –

t ∫ = – log | t | + c Putting t = 1 + cos

sin x

31. 2

(1 + cos)

x – sin x

sin

x dx

Sol. Let I = 2 (1 cos

) + ∫ ...(i) (1 cos)

dx = – 2

x

+ x ∫

dx

Put 1 + cos x = t. Therefore – sin x =

t – –

⇒ – sin x dx = dt

dt

dt

∫ dt = 1

t –

t ∫ = – 2

1 + cot x

∴ From (i), I = – 2 1 cot + x ∫ x

= 1

dx = 1cos

x

t + c = 1

1 sin

1 cos + x +

c. 32.1

∫

dx = 1

_ + c 1

dx

∫

x x

+

sin cos

Sol. Let I = 1 □ □ □ □

□ + sin x

dx = 1 2sin x x +∫ dx = 1 22

=sin sin sin x

x x +∫ x x

+ +∫ dx sin cos

x

sin cos

sin cos x x Adding and subtracting cos x in the

numerator of integrand, xxxx I = 1 2sin cos cos sin

+−+

+∫ dx

x x

sin cos

xx xx

= 1 2(sin cos) (cos sin)

+−−

+∫ dx

x x

sin cos

xx xx

= 1 2sin cos (cos sin)

+ − − □ □□ □

□ + +∫ dx

ab a b

xx xx

x x

sin cos sin cos

x x

dx dx

= 1 2 (cos sin) 1 sin cos

= 1 2 cos sin 1 sin cos

x x

− − □ □□ □

□ +∫

= 1 2[x – I1] ...(i) x x

dx

− = − □ □

□ □ □ □ ∵ c − −□ □ + ∫ ∫

□ □ □ □

cc

dx =cos sin
x x

where I 1

+ cos x = t dt

–

+∫
x x
sin cos
Put DENOMINATOR sin x dt
∴ cos x – sin x = x) dx = dt ∴ I_1 = t∫
dx ⇒ (cos x – sin
= log | t | = log | sin x + cos x |.

Note. Alternative solution for finding I_1

I 1 =cos sin
x x
–
dx = log | sin x + cos x | +∫
sin cos
x x

′ =
f x dx f f x
x

∵ ∫( ) log| ( )|( )

Putting this value of I_1 in (i), required integral = 1 2
[x – log | sin x + cos x |] + c.

33. 1
1 – tan x
1 tan – x∫ x
–∫ dx = 1 x
1sin 1
Sol. Let I = 1 dx = cos x x

dx cos

∫ − x

cos sin x

x x − ∫

+

= cos

dx = 1

2 2 cos

dx = cos dx

x

x x − ∫ 1

2 cos − ∫

cos sin cos sin x x

x x

cos sin

Subtracting and adding sin−++

x in the Numerator, cos sin

xx xx

= 1/2 cos sin sin cos

xxxx

= 1/2 cos sin sin cos

dx = 1/2 sin cos

− ∫ x x x x

dx

1 cos sin

+ − − ∫

− + xx xx

+ dx

cos sin cos sin + − ∫ x x

dx dx

x x

= 1/2 sin cos − − − ∫ ∫ f x dx f x

f x

1 cos sin − x x

= 1/2 [x – log | cos x – sin x |] + c ( ) − ∫

cos sin

log| ( )| ′ = ∵ ∫ x x

( ) denominator cos x – sin x = t.

Note. Alternative solution for

evaluating sin cos x x

− − dx, put

Sol. Let I = tan

x

sin cos
x x

x

34. tan

x

dx = x
x x
dx

tan

x x∫ ∫

x ∫ dx ...(i)
t 1 x

sin cos
sin cos cos

cos x

x x ∫ dx =

^{2}sec
x
tan

Put tan x = t. tan ∵ t t

= 2
tan cos ∴ sec^2

dt

x = dt ∴ From (i),

dx ⇒ sec$_2$ x dx =

t∫ =$^{1/2}$ dt = $_{1/2}$

t + c = 2 t

∫ ^{2}(1 + log)

+ c = 2 tan c. 1 2

35. I = x

x x +

dt

^{2}(1 log) x

t$^-$

Put 1 + log x dx ...(i)
= t x

Sol. Let I = +∫

dt
dx

∴1
x =
dx ⇒ x = dt
t ∫ dt = 3 3

∴ From (i), I = 2 t + c = 1 3(1 + log x)^3 + c.

36. 2(+ 1)(+ log) xx x
x

Sol. Let I = 2(1)(log) xx x
+ + ∫ dx ...(i)
x

Put x + log x = t

∴ 1 + 1
x =
dt
dx ⇒ x 1 + = dt dx = dt

dx ⇒ x 1

t dt ∫ = 3 3

Putting t = x + log x, 1 3(x + log x)^3 + c. 3 14

37. ⁻ x x
x
x
sin(tan)

t +

1 + x^8

∴ From (i), I = 2

dx = 1 4^3 - + ∫

3 1 4 x x

x^8

4 sin (tan) . 1^x dx ...(i)

sin (tan) – 1 4

+∫

Sol. Let I Put (tan^{-1} x

x^4) = t

= 1 x 8

[Rule for sin (( )) ( ) ; put ( ) f x f x dx f x t ′ = ∫]

dx f x dx 1

dt d d f x f x

) 1 (( )) x^4=

1 tan ( ) (dx $dx^1{}_2$

dx =

d =

∴ 4 2 1() + x 4 + ∵

x

dt_8

1

(i), ⇒

∴ From 3 + x

t∫ $= \frac{1}{4} - \cos(\tan^{-1} x^4) + c.$

Choose the

$I = \frac{1}{4}\sin$ $dt = -\frac{1}{4}\cos t + c$

correct answer in Exercises 38 and 39:

38. ∫ 9 10

x

10 +10 log 10

x dx

x

+10

e

x equals

x – x^{10} + C (B) 10

x + x^{10} + C (C) (10

$^{x} - x^{10})^{-1}$ + C (D) log (10

(A) 10 $+ ^{e}$

$+ x^{10}$) + C.

Sol. Let I =

$+\int$ dx ...(i)

x

x

$x^{9\ x}$ $x^{10}10$

10 10 log 10

Put x^{10} + 10

^{x} = t

d aa a

dx   = ∵ dt

∴ $(10x^{9} + 10$ log

^{x}log^{e} 10) dx = dt( ) + c

Putting t = x^{10} + 10

x x

e ∴ From (i), I =

t$\int$ = log | t |

^{x}, I = log | x^{10} + 10

$^{x} + x^{10}$) + c. correct answer. OR

or I = log (10 ^{x}| + c

∴ Option (D) is the

$x^{9\ x}$

10 10 log 10 f x

$+\int$ dx =( )

+ 10 x

10 e

x

$\int'$ dx = log |x

( )

f (x) | + c f

. 20

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$_{x}$| + c∴ Option (D) is the correct answer.

= log | x^{10} + 10

dx

39.$\int^{2\ 2}$sin cos

x x equals

(A) tan x + cot x + C (B) tan x – cot x + C (C) tan x cot x + C (D) tan x – cot 2x + C.

x x $\int$ =$^{2\ 2}$

dx

Sol.$^{2\,2}$sin cos x x

sin cos

x x

sin cos

x x

sin cos

$+\int$

+= +

dx [∵1 = $\sin^2$ x + $\cos^2$

x] 2 2

2 2

dx

ab a b

+ $\int$

cos sin x x

=22 22

xx xx

sin cos sin $=^2$sec

cos

x$\int$

dx + 2

cosec dx =

x$\int$

1 1

+ $\int$

dx = 2 2 (sec cosec

tan x – cot x + c∴

Option (B) is the correct answer.

) x x$^+\int$ = 2 2 dx

c cc

Exercise 7.3

Find the integrals of the following functions in Exercises 1 to 9: 1. $\sin^2(2x + 5)$

Sol.^{2}sin (2 5) dx = 1 2(2 5)) 2− x$\int$

x$^+\int$ (1 cos + dx

1 sin (1 cos 2) ; put 2 5 2 x θ= − θ θ= + ∵2

= $\frac{1}{2}$(1 cos(4 10)) − +

= $\frac{1}{2}$sin (4 10)

dx = $\frac{1}{2}$ 1 cos (4

10) dx x dx − + $\int\int$

x$\int$

x

x

4 Coeff. of

$+ c = \frac{1}{2}x - \frac{1}{8}\sin(4x + 10) + c.$

2. sin 3x cos 4x

Sol. $\int \sin 3x \cos 4x \, dx = \frac{1}{2}\int 2 \sin 3x \cos 4x \, dx$

$= \frac{1}{2}\int (\sin(3x + 4x) + \sin(3x - 4x)) \, dx$

[∵ 2 sin A cos B = sin (A + B) + sin (A – B)]

$= \frac{1}{2}\int (\sin 7x + \sin(-x)) \, dx = \frac{1}{2}\int (\sin 7x - \sin x) \, dx = \frac{1}{2}\left[\int \sin 7x \, dx - \int \sin x \, dx\right]$

$= \frac{1}{2}\left[\frac{-\cos 7x}{7} - (-\cos x)\right] + c = -\frac{1}{14}\cos 7x + \frac{1}{2}\cos x + c.$

3. cos 2x cos 4x cos 6x

Sol. $\cos 2x \cos 4x \cos 6x = \frac{1}{2}(2\cos 6x \cos 4x)\cos 2x = \frac{1}{2}[\cos(6x + 4x) + \cos(6x - 4x)]\cos 2x$

[∵ 2 cos x . cos y = cos (x + y) + cos (x – y)]

$= \frac{1}{2}(\cos 10x + \cos 2x)\cos 2x = \frac{1}{4}(2\cos 10x \cos 2x + 2\cos^2 2x) = \frac{1}{4}[\cos(10x + 2x) + \cos(10x - 2x) + 1 + \cos 4x]$

$= \frac{1}{4}(\cos 12x + \cos 8x + \cos 4x + 1)$

$\therefore \int \cos 2x \cos 4x \cos 6x \, dx = \frac{1}{4}\int (\cos 12x + \cos 8x + \cos 4x + 1) \, dx$

$= \frac{1}{4}\left[\int \cos 12x \, dx + \int \cos 8x \, dx + \int \cos 4x \, dx + \int 1 \, dx\right] =$

1
4sin 12 sin 8 sin 4

xxx + c.

x

+++

12 8 4

Note. We know that $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

$\therefore 4 \sin^3 \theta = 3 \sin \theta - \sin 3\theta$

Dividing by 4, $\sin^3\theta = \frac{3}{4} \sin \theta - \frac{1}{4}\sin 3\theta$ ...(i) Similarly, $\cos^3\theta = \frac{3}{4} \cos \theta + \frac{1}{4} \cos 3\theta$ ...(ii) $[\because \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta]$

4. $\sin^3(2x + 1)$

Sol. To evaluate $\int \sin^3 (2x + 1)\, dx$

We know by Eqn. (i) of above note that $\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$

Putting $\theta = 2x + 1$, we have

$\sin^3(2x + 1) = \frac{3}{4} \sin (2x + 1) - \frac{1}{4}\sin 3 (2x + 1)$

$= \frac{3}{4} \sin (2x + 1) - \frac{1}{4}\sin (6x + 3)$

$\therefore \int \sin^3 (2x + 1)\, dx = \frac{3}{4}\int \sin (2x + 1)\, dx - \frac{1}{4}\int \sin (6x + 3)\, dx$

$-\frac{1}{4}\cos (6x + 3)$

$= \frac{3}{4}\cos (2x + 1)$ ... + c

2 of x

6 Coeff.

$= -\frac{3}{8} \cos (2x + 1) + \frac{1}{24}\cos (6x + 3) + c.$

OR

To integrate $\sin^n x$ where n is odd, put $\cos x = t$.

$\therefore \int \sin^3 (2x + 1)\, dx = \int \sin^2 (2x + 1) \sin (2x + 1)\, dx$

$= \frac{1}{2} \int -2 [1 - \cos^2 (2x + 1)] (-2 \sin (2x + 1))\, dx$ ...(i)

Put $\cos (2x + 1) = t$

$\therefore -\sin (2x + 1) \frac{d}{dx} (2x + 1) = \frac{dt}{dx}$

$\therefore -2 \sin (2x + 1)\, dx = dt$

$\therefore$ From (i), the given integral $= \frac{1}{-2}\int (1 - $

$= \frac{1}{2}\int (t^2 - 3)\,dt$. 23

$+ c = \frac{1}{2}t^3 - t$

$t + \frac{1}{6}t^3 + c$

= $\frac{1}{2}$ − cos (2x + 1) + $\frac{1}{6}$cos^3(2x + 1) + c.

5. $\sin^3 x \cos^3 x$

Sol. $\int \sin^3 x \cos^3 x\,dx = \int (\sin x \cos x)^3\,dx$

$= \frac{1}{8}\int (2\sin x \cos x)^3\,dx = \frac{1}{8}\int \sin^3 2x\,dx$

$= \frac{1}{8}\int \frac{3\sin 2x - \sin 6x}{4}\,dx$

Putting $\theta = 2x$ in $\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$

$= \frac{3}{32}\int \sin 2x\,dx - \frac{1}{32}\int \sin 6x\,dx$

$= \frac{3}{32}\left(\frac{-\cos 2x}{2}\right) - \frac{1}{32}\left(\frac{-\cos 6x}{6}\right) + c = -\frac{3}{64}\cos 2x + \frac{1}{192}\cos 6x + c.$

OR

To evaluate $\int \sin^3 x \cos^3 x\,dx$, Put either sin x = t or cos x = t. (The form of answer given in N.C.E.R.T. book II can be obtained by putting cos x = t)

6. sin x sin 2x sin 3x

Sol. sin x sin 2x sin 3x = $\frac{1}{2}$(2 sin 3x sin 2x) sin x = $\frac{1}{2}$[cos (3 x–2x) – cos (3x + 2x)] sin x

[∵2 sin x sin y = cos (x – y) – cos (x + y)]

= $\frac{1}{2}$ (cos x – cos 5x) sin x = $\frac{1}{4}$ (2 cos x sin x – 2 cos 5x sin x) = $\frac{1}{4}$ [sin 2x – {sin (5x + x) – sin (5x – x)}]

[∵2 cos x sin y = sin (x + y) – sin (x – y)]

= $\frac{1}{4}$(sin 2x – sin 6x + sin 4x)

∴ sin sin 2 sin 3 xxx ∫ dx = $\frac{1}{4}$(sin 2 sin 4 sin 6) xxx + – ∫ dx

= $\frac{1}{4}$ sin 2 sin 4 sin 6 x dx x dx x dx + – ∫∫∫

= $\frac{1}{4}$cos 2 cos 4 cos 6

−−+

+

xxx c. 246

7. sin 4x sin 8x

Sol. sin 4 x x ∫ sin 8 dx = $\frac{1}{2}$2 sin 4 x x ∫ sin 8 dx

dx

= $\frac{1}{2}$[cos (4 8) cos (4 8)] xx xx −− + ∫

[∵2 sin A sin B = cos (A – B) – cos (A + B)]

= $\frac{1}{2}$(cos (4) cos x x ∫ 12) − − dx = $\frac{1}{2}$(cos 4dx [∵ cos 12) x x − ∫ cos (– θ) = cos θ]

= $\frac{1}{2}$ cos 4 cos 12 x dx x dx − ∫ ∫ = $\frac{1}{2}$sin 4 sin 12

x x −

+ c.

4 12

x

x

8. 1 – cos

1 + cos

x

Sol. 1 cos 2

dx = 2 tan 2 dx x

– 2 sin 2 x ∫ x ∫ x

dx =

+ ∫

1 cos x 2 2 cos 2 dx (∵ $\tan^2\theta = \sec^2\theta - 1$)

= 2 sec 1 2

θ θ − θ= + θ= ∵

2 2 1 cos 2 sin and 1 cos 2 cos x – ∫

2 2

= 2 sec 2 – x + c = 2tan 2

dx

Coeff. of dx – 1 ∫ = tan 2 1

x ∫ x x – x + c.

+ x ∫

9. cos 2 x → x x x

1 + cos

Sol. cos dx

1 cos

Adding and subtracting 1 in the numerator of integrand,

x

= 1 cos 1 + –

+ – dx

+ ∫ 1 cos 1

– = + + ∫ dx =

x c cc

– ∵ ab a b

1
1

1 cos x x x 1 cos 1 cos

dx – x∫

∫ = dx = 1∫

$1 \quad 2 \sec^2 \frac{x}{2}$

2 cos $^2\frac{x}{2}$

x

= x – $\frac{1}{2}$ tan $\frac{x}{2}$ + c = x – tan $\frac{x}{2}$ + c.

1

2

Find the integrals of the functions in Exercises 10 to 18: 10. $\sin^4 x$

Sol. 4 sin (sin) x∫ cos 2 –

x∫ dx = 2 1 x 2

dx = 2 2 dx

$^2(1 \cos 2)$

dx x∫ 1

– dx = $4^2(1 \cos 2\, 2$ x x∫

= cos 2) + –

x

4 +

x

= $\frac{1}{4}$ 1 cos 4 1 2 cos 2 + θ θ = ∵ + –

∫ dx 2 1 cos 2 cos 2 2

 = $\frac{1}{4}$ 2 1 cos 4 4 cos 2

++ – dx = $\frac{1}{8}$(3 cos 4 x x∫

x x ∫ 4 cos 2) + – 2 dx

= 1/8 3 1 cos 4 4 cos 2 dx x dx x dx + − ∫∫ ∫

= 1/8 sin 4 4 sin 2 3 4 2

x x

x + − + c = 3/8 x + 1/32 sin 4x − 1/4 sin 2x + c 11. $\cos^4$ 2x

Sol. $\int \cos^4 x\, dx = \int (\cos^2 x)^2 dx$

2 1 cos 4

dx = (1/4) ∫ (1 + cos 4x)² dx = 1/4 ∫ (1 + 2 cos 4x + cos² 4x) dx

= 1/4 ∫ (1 + 2 cos 4x + (1 + cos 8x)/2) dx ∵ $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

= 1/4 ∫ (1 1 cos 8 / 2 + 1 2 cos 4) dx

= 1/4 ∫ (2 1 cos 8 4 cos 4 / 2) dx = 1/8 ∫ (3 cos 8 4 cos 4) x x dx

2 1 cos 2 cos 2 x dx

= 1/8 3 1 cos 8 4 cos 4 dx x dx x dx + + ∫∫ ∫

= 1/8 sin 8 4 sin 4 3 8 4

x x

1 + cos x

+ x ∫ dx = 2 1 cos 2 sin

x + + + c = 3/8 x + 1/64 sin x

8x + 1/8 sin 4x + c 2 sin x

12.

Sol. $\int \frac{1+\cos x}{1-\cos x} dx$ = ∫ (1 cos)(1 cos) x / − − + x dx

1 cos 1 cos x 1 cos x

= (1 cos) − x ∫ dx = 1 ∫ dx − cos x ∫ dx = x − sin x

+ c. Note. It may

be noted that letters a, b, c, d, ..., q of English Alphabet and letters α, β, γ, δ of Greek Alphabet are generally treated as constants.

13. cos 2 – cos 2

x

α

cos –

α

cos x

– α ∫ dx = 2 2 (2 cos 1) (2 cos 1)

x

Sol. cos 2 cos 2

– α x – – α– x

x

cos cos

– α ∫ cos

dx cos

– α ∫ dx = 2 2 2 cos 2 cos 2 2 2 cos 1 2 cos 1 x – – α+ x

– α

=

cos cos x – α ∫ dx cos cos

2 2 cos cos

x – α x x x

x

– α ∫ dx = 2 (cos cos)(cos cos) –α +α

= 2 – α dx

cos cos x ∫ x (cos cos)

sin x + 2x cos α + c.

= 2 (cos cos) x + α ∫

dx = 2 cos cos x dx dx +

α ∫

. 26

dx] = 2 [sin x + (cos α) x] + c

∫ = 2 [sin x + cos α ∫ 1

= 2

a ∫

Class 12 Chapter x sin a.

7 - Integrals dx =

dx = sin a 1 ∫

Remark. sin

Please note that sin

14. cos – sin
x x
1 + sin 2
a ∫
dx ≠ – cos a.

sin
x x
–

cos sin
x –

x x
+ + ∫
dx

Sol. Let I = cos

dx = 2 2

+ ∫

1 sin 2
x
cos sin 2 sin

cos
x x + ∫ = 2
cos sin

dx ...(i)

x x xx

Put cos x + sin x = t. dt

–

(cos sin) x x

dx . Therefore (cos x – sin x)

dx = dt. dt = 1 1

∴ – sin x + cos x = dt

t –

t – ∫ + + c.

t ∫ = 2

∴ From (i), I = 2

cos sin x x

15. $\tan^3 2x$ sec

⇒ I = 1

2x – + c

t – + c = 1

–

2 sec 2
Sol. Let I = ^{3}tan 2
x xx ∫

dx = ^{2}(sec 2 1)

x ∫ sec 2x dx = 2 tan 2 tan

= 1 2^2(sec 2 1)(2

sec 2 tan 2 x xx – ∫

dx [∵ $\tan^2\theta = \sec^2\theta - 1$]

sec 2 tan 2) x dx ...(i)

xx – ∫

sec 2x tan 2x

d

Put sec 2x = t. Therefore dtdx ∴ 2

dx (2x) =

sec 2x tan 2x dx = dt ∴ From dt = 1 2

(i), I = 1 2 2(1) t ∫ ( )2

t dt dt − 1 ∫ ∫

t + c

t3 _ 1 2

t

= 1 2 3 3 (−) ( ) + c = 1 6

t

Putting t = sec 2x, = $\frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + c$.

16. $\tan^4 x$

Sol. $\int \tan^4 x\, dx = \int \tan^2 x \tan^2 x\, dx = \int \tan^2 x (\sec^2 x - 1)\, dx = \int (\tan^2 x \sec^2 x - \tan^2 x)\, dx = \int \tan^2 x \sec^2 x\, dx - \int \tan^2 x\, dx = \int \tan^2 x \sec^2 x\, dx - \int (\sec^2 x - 1)\, dx$

$= \int \tan^2 x \sec^2 x\, dx - \int \sec^2 x\, dx + 1 \int dx$

For this integral, put tan x = t.

∴ $\sec^2 x = \frac{dt}{dx}$ or $\sec^2 x\, dx = dt$

$= \int t^2 dt - \tan x + x + c = \frac{t^3}{3} - \tan x + x + c$

Put t = tan x, = $\frac{1}{3}\tan^3 x - \tan x + x + c$.

17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Sol. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} \right) dx$

ab a b c cc

x x

+ = + ∵

= 2 2 cos sin

x x

+ ∫ dx xx xx

+ ∫ dx = sin cos cos cos sin sin

x x

sin cos

dx

= (tan sec cot

cosec) xx x x + ∫

x x ∫ x x ∫ cosec x + c.2

= sec tan x x dx + cosec cot dx = sec x –

x 2

cos 2 + 2 sin + ∫

18.

2 2

dx = cos 2 2

sin

cos

2

− + ∫ dx x x

Sol.

x x 2

(1 2 sin) 2 sin

cos x

x = 2 1 cos 2

cos x ∫ dx = 2 sec Note. Method to x ∫

Integrate the dx = tan x + c.

functions in evaluate 1

Exercises 19 to 22:

p q

x x ∫

dx if (p

+ q) is a

sin cos

negative even integer (= – n (say)); then multiply Numerator and

n x.

Denominator of integrand by sec

1

19. 3

sin cos x x

1

sin cos x x ∫ dx ...(i)

Sol. Let I = 3

Here p + q = – 1 – 3 = – 4 is a negative even integer. So multiplying both Numerator and Denominator of integrand of (i) by $\sec^4 x$,

4

xxx∫ = sec

dx

sec

3 4

4

I =

x x tan

sin cos sec

x ∫

1 sin

sin cos sec sin ∵ 34 3

dx

= = =

x

cos . tan

4

xxx xx x

x x

2 2 sec sec cos cos

sec

x x

or I =

x x

+∫

dx = 2 2 (1 tan) x ∫

dx ...(ii)

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tan

Put tan x = t

tan

x

dt

dx ⇒ $\sec_2$ x dx

dt = 2 1 t

∴ $\sec^2$

+

x =

= dt ∫

∴ From (ii), I =

=1 + ∫ dt t t

t

^{2}(1) t t

t∫ dt +

t+ c

dt = log | t | + 2 2

t

t∫

+

∫ dt =1

Putting t = tan x, = log | tan x | + $\frac{1}{2}\tan^2 x$ + c.

cos 2 x

20. 2

(cos + sin)

x x

x x + ∫ dx = 2 2 2

cos 2 x

x x cos sin −

Sol. Let I = 2 (cos sin) x xx x

+∫ dx x x

(cos sin)

=(cos sin)(cos sin) + +∫

dx = cos sin

x x

+ −

−

x xx x

(cos sin)(cos sin)

+∫ dx ...(i) cos sin

Put DENOMINATOR cos x + sin x = t

x x

dt

∴ – sin x + cos x = dx ⇒ (cos x – sin x)

dx = dt

dt

∴ From (i), I = ∫ t

= log | t | + c = log | cos x + sin x | + c Note. Another method to evaluate integral (i) is, apply f x

( )

′

∫ dx = log | f (x) |.

f x

( )

21. $\sin^{-1}$(cos x)

sin 2 x − π

∫

dx = 1 sin

Sol. $\sin^{-1}$ (cos) x − = 2 x − ∫ dx

π

∫ dx – x ∫

dx = $\frac{\pi}{2}$ ∫ dx

−

= $\frac{\pi}{2}$ 1 ∫ dx = $\frac{\pi}{2}$

x − $\frac{x^2}{2}$ + c.

22. 1

dx − 1 x ∫

cos (–) cos (–) xa xb

Sol. Let I = 1

cos ( ) cos ( ) xa xb – – ∫ dx ...(i) Here (x – a) – (x – b) = x – a – x + b = b – a ...(ii) By looking at Eqn. (ii), dividing and multiplying the integrand in (i) by sin (b – a),

sin [( ) ( )]

–––

I = 1

sin ( ) b a −

– – ∫ dx [By (ii)]

xa xb

cos ( ) cos ( )

= 1

sin ( ) b a −

xa xb xa xb

sin ( ) cos ( ) cos ( ) sin ( ) −

–– – –

= 1

sin ( ) b a − b a

sin ( ) – – ∫ dx xa xb

–

cos B – cos A sin B] . 29

– – ∫ dx

xa xb

cos ( ) cos ( )

xa xb

= 1

xa xb xa xb

sin ( ) cos ( ) cos ( )

sin ( ) b a − = 1

sin ( ) xb xa xb

cos ( ) cos ( ) cos ( )

cos ( ) A–B A B

= –

C CC

–– –– –

–– ––

∫

dx

xa

cos ( ) cos ( )

[∵ sin (A – B) = sin A

( )] xa xb –– – ∫ = 1

sin ( ) b a − [tan ( ) tan dx

x ∫

sin ( ) b a – [– log | cos (x – a) | + log | cos (x – b) |] + c (∵ tan dx = – log | cos x |)

x b

sin ( ) b a – log cos ( )

= 1

–

+ C. log log log m m n

–

n

x a

cos ( )

–∫ dx

2 2

x x

sin cos

=

sin cos

22 22 xx xx

–

∫ dx

ab a b x x

Sol.

Choose the correct answer in Exercises 23 and 24: 23. ∫ 2 2 sin – cos x x dx is equal to x x 2 2 sin cos

(A) tan x + cot x + C (B) tan x + cosec x + C (C) – tan x + cot x + C (D) tan x + sec x + C

sin cos sin cos 1 1 – = –

∵ c cc

2 2

x x

sin cos

2 2

dx

–

∫ cos dx = 2 2 (sec

= 2 2

sin x x

cosec) x x –∫

x ∫ x ∫ (– cot x) + cot x +

dx – 2 cosec dx = tan x – C = tan x +

$=^2$sec

C ∴ Option (A) is the correct answer. x

24. ∫ $_2$(1 +)

e x

dx equals

e x

x

cos ( )

^{x}) + C (B) tan (xe^x) + C (C) tan (e

^{x}) + C (D) cot (e

(A) – cot (ex + C x ^{x})

e x

\+ x

) ∫

x

Sol. Let I

= $_2$(1 dx ...(i) e

cos ( )

^{x}. x = t

Put e

[To evaluate ∫ (T-function or Inverse T-function f (x)) f ′(x) dx, put f (x) = t]

dt

^{x}. 1 + xe

Applying ^{x}= t ∫

Product Rule, =2cos

e ^{x}(1 + x) dx

= dt dx

or e dt

∫= ^{2}sec dt

∴ From (i), I t

^{x}) + C ∴Option (B) is the correct answer. . 30

= tan t + C = tan (x e

Exercise 7.4

Integrate the following functions in Exercises 1 to 9:

x^2 x

1\. 6 x + ∫ dx

3 $x^2$3

3 Sol. Let I = 6

+1 Put x^3 = t

$=^2$ x + ∫ dx ...(i) 32 2

1
() 1
x

dx ⇒ $3x_2$ dx
= dt dt

∴ $3x^2$=
dt

t + C

∴ From (i), I = 2 2 1

t + ∫ = 1 1 $\tan^{-1}$ 1

x a a a

x dx

∵ ∫ 1 1 = $\tan^{-1}$

2 2

+ C.

Note. $ax^2 + b$ (a ≠ 0) is called a pure quadratic. 1

1 4 + x

2. 2
+

1 1

Putting t = x^3; = $\tan^{-1}(x^3)$

1 4 + x ∫ dx = 2 2

Sol. Let I = 2 1 ∫ dx = log 2 2 x xa +

dx (2) 1 x +

+ , x a +

∫

Using 2 2

2 2 log (2) (2) 1
x x
+ +

I =

+ C = $\frac{1}{2}$ $\log^2$ 2 41 x x + + + C.

→

x

(2 –) + 1 x

3. 2

dx = log 2 2 x xa + + ,

x a +

2 2 log (2) (2) 1

x x

−+ − +

=

− → + C

1

1

2 Coeff. of 1

(2) 1 − + x ∫ dx = 2

Sol. Let I = 2 1(2) 1 − 1 Coeff. of x

+ x ∫

dx = − log 2 2 4 41 −+ + − + x xx + C

∫

Using 2 2

= log 2 1

2 45 −+ − + xx x

+ C.

m n mn nm

n m

log (log log) log log log

1

4. 2 9 – 25x

− =− − = − = ∵ . 31

1

1

9 25 − x ∫ dx = 2 2

Sol. Let I = 2

x

3 (5) − x ∫ dx

x dx a

1 = sin

□ □ a x

1
5 sin 3 –

→ + C⁻¹

∵ ∫

□ □

=

x

–

□ □

1
5 sin⁻¹ 5 3

2 2

C.

x

3

5 Coeff. of =

x □ □

□ □ +

□

5. 4

x x

1+2

x 3 2

+ x ∫ dx = 3 2 2 2

Sol. Let I = 4 1 2

+ x ∫ dx ...(i) 1 2( )

Put x^2 = t. ∴ 2x = dt

1 dt

dx ⇒ 2x dx = dt

∴ From (i), I = 3 2^2 1 2

+ t ∫ = 3 2 2 2 (2) 1 t + ∫ dt

1 2 1 tan⁻
t

= 3 2

→ + C = 3

1 1

2 Coeff. of

Putting t = x^2, = 3

t

2

x

2 2tan− 1(2

t) + C

2 2tan− 1(2 x2) + C.

x 2

6. 6 1 –

− x∫ dx = 23 2 1()

x

x

− x∫ dx = 1 32 3 2

Sol. Let I = 3 x

6 1

− x∫ dx 1()

t

Put x^3 = t. Therefore

dt

dx ⇒ 3x2 dx = dt. 1

3x^2= dt t∫ = 1 32 2

∴ I = 1 32 1

+

2 1 · log 1 1

t

– + C

1 − t∫ dt = 1 31

∵ ∫

dx

a x a a–x

a x

1 1+ = log– 2

Putting t = x^3, = 1 6 log3 3

2 2

1 + x

1

7. 2 − 1 – 1 x

– + C. x

x

–

Sol. Let I = $_2{}^1 1$ x

– – ∫

–

x

– ∫ dx = $_{2\,2}1$

x

dx

1 1

x x

. 32

1

x

dx –$_{2\,2}$

x – ∫

$=\frac{1}{2}$ $=\frac{1}{2}$ $_2{}^2 1$ x dx

x –

1 ∫

∵ ∫$^{2\,2}$

x – ∫

dx – log $^{2\,2}$ x x + − 1 ...(i) dx x x a

1 = log| + – |

x

$^{2\,2}$

x a –

Let I_1 = $_2{}^2 1$

dx

dt =

dt

x – ∫

Put x^2 – 1 = t. Therefore dt

2x = 1/2

dx ⇒ 2x dx = dt∫ 2

∴ I 1 = t∫ =1/2 t−

t = 2 + C 1

t= 2 2 x − 1

Putting this value of I_1 = x − ∫

2 2 1 dx in (i),

x

I = 1 2(22 x − 1 + C) – log | x +2 x − 1 | =2 x − 1

+ C 2– log | x +2 x − 1 |

=2 x − 1 – log | x +2 x − 1 | +

x

t. Therefore $3x^2$= ∴

8. 6 6 + x a

From (i), I = 1 3 2 6

∫

dx = 1 32

Sol. Let I =

dt

dx ⇒ 3x2 dx = dt.

x23 x dx ...(i)32 6

C_1 where C 1 = C 2·

2

( ) 1

x a +

6 6 x a +∫ Put x^3 =

∫= 1 32 32

dt

t a + ( ) ∫ dt

t a +

+ C

= 1 3log 2 32

tt a + + ( )

1 dx x x a log

∵ ∫2 2

= ++

2 2

+

x a

Putting $t = x^3$, $= \frac{1}{3}\log\left|x^3 + \sqrt{x^6 + a^6}\right| + C.$

sec 2
2
tan + 4 x
2

x

9.

sec

x

Sol. Let I = $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}\, dx$ ...(i)

. 33

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Put $\tan x = t$. $\therefore \sec^2 x = \frac{dt}{dx} \Rightarrow \sec^2 x\, dx = dt$

$\therefore$ From (i), $I = \int \frac{dt}{\sqrt{t^2 + 2^2}} = \log\left|t + \sqrt{t^2 + 2^2}\right| + C$ $\because \int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right|$

Putting $t = \tan x$, $I = \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C.$

Integrate the following functions in Exercises 10 to 18: Note. Rule to evaluate

  = ++ +

dx or ∫ 1

Quadratic

dx or ∫Quadratic dx

∫1 Quadratic

Write Quadratic. Take coefficient of x^2 common to make it unity. Then complete squares by adding and subtracting 2 1 coefficient of

Sol. $_2$1

2x

    x x +2 +2

10. $_2$1

dx =

x x + + 2 2 ∫ $_2$1 1

x x + ++ 2 11 ∫ dx

dx = 2 2 ∫

1

dx = log | x + 2 2 x a +|

∫

x a +

Using 2 2

(1) 1 x + +

= log | x + 1 + 2 2 (1) 1 x + + | + c = log| x + 1 + 2 x x + + 2 2 |+ c. 11. $_2$1

9 +6 +5 x x

Sol. Let I = $_2$1

9 65 x x + + ∫ dx ...(i) 1

Quadratic ∫ dx

Here Quadratic expression = $9x^2 + 6x + 5$

Making coefficient of x^2 unity, = 9 2 6 5

x x

 + +

9 9

= 9 2 2 5

x x

3 9

$_2$ 1 Coefficient of

To complete squares, = 9^2

adding and subtracting

2^x

3 3 99

= 2 2 12 9 x

x

1 1 23 3

. 34

$_2$ 2 1 15 ⇒ $9x^2$ +

3 9 x

6x + 5 = $9^{2\,2}$ 1 2

= 9

∫ dx

^{2} 1 4 3 3 x

1

Putting this value in (i), I = $_{2\,2}$ 1 2 9 3 3 x

1

= $^1 9^{2\,2}$

∫ dx

1 2

x

∵

= ^{1} 9 ^{1} 2 3 3 x + 1 –

∫

  tan−1 3 3 2 3

x dx + c 1 2 2

1 1 tan

x a a a

x + 3 1

3

x + + c.

$= \frac{1}{9} . \frac{3}{2} \tan^{-1}$

+ c = $\frac{1}{6} \tan^{-1}$ 3 1

2

2

3

1

12. 2

7–6 – x x

1

7 6 – – x x ∫ dx ...(i) 1 Type Quadratic ∫ dx

Sol. Let I = 2

Here Quadratic expression is 7 – 6x – x^2 = – x^2 – 6x + 7. Making coefficient of x^2 unity, = – (x^2 + 6x – 7).

To complete squares, adding and 2 1 coefficient of 2x

subtracting 2 1 6

= 9 2

=

.

= – [x^2 + 6x + 9 – 9 – 7] = – [$(x + 3)^2$ – 16] ...(ii) = – $(x + 3)^2$ + 16 = 4^2 – (x + $3)^2$...(iii) (Note. Must adjust negative sign outside Eqn. (ii) in the bracket as shown above because otherwise we shall get − 1 = i on taking square roots.]

Putting the value of quadratic expression from (iii) in (i), x

+ + c

4 (3) − + x ∫ 4

1 dx = sin− 1 3

I = 2 2

$\because \int^{1}$ = 2 2

–

1 sin

13. 1

x dx a

a x –

(– 1)(– 2) x x

(1)(2) x x – – $\int$ dx = 2 1

1 Sol. Let I =

x xx – –+ 2 2 $\int$ dx. 35

1

x x – + 3 2 $\int$ ...(i)

= 2

Here quadratic expression is $x^2 - 3x + 2$. Coefficient of x^2 is already unity. To complete squares, adding and subtracting 2 1 coefficient of

2 x 3 2 , 2 3 2 3 2

i.e., 2

–

=

$x^2 - 3x + 2 = x^2 - 3x + {}^2 3$ – 1 4 9 98 1 2

9 – 4 + 2 2

– – + –

= x – – 2 1 2

+= = $\because$

2

4 44

x^2 3 2 ...(ii) 1 $\int$

dx

=

Putting this value in (i), I = 2 2 3 1

– –

2 2 x

2 2 3 31

= log

2 22 x x + c

–+ – –

1 = log + –

2 2 x a –

dx x x a

∵ ∫ 2 2

= log3 2 3 2

2 x xx –+ – + + c. [By (ii)] 1

14. 2

8+3 – x x

1

dx ...(i)

8 3 + –

x x ∫

Sol. Let I = 2

Here quadratic expression is $8 + 3x – x^2 = – x^2 + 3x + 8$. Making coefficient of x^2 unity, = – (x^2 – 3x – 8).

To complete squares, adding and subtracting

2 x

2 2

= 2 3 2

2 1 coefficient of

$8 + 3x – x^2 = –$ 2 3 9 8

2 2 x x

2 4 x

– –

–+ – –

= 41 4 – 2 3 2

2

= –

3 3 3 8

– x

2 4 x 22 3

– –– = –2 3 41

(See Note given in the solution of Q.N. 12) 41 . 36

– ...(ii) 1

=

– 2 2 x

dx

∫

Putting this value in (i), I = 2 2 – –

41 3

x

= sin– 1 2 3 41

1

= sin– 1 x – 2 2

3 2 2

x dx a (–)(–) x ax b

1 sin –

a x

+ c 1 ∵ ∫ 15.

41

2

=

2 x – + c.

ax b – –

∫

dx =

2 1

( )( ) x

Sol. Let I = 1 1 ∫

dx

x bx ax ab ––+ x x a b ab – ++ ( ) ∫ ...(i)

= 2

Here Quadratic expression = $x^2 – x(a + b) + ab$ 2 1

coefficient of

=2

2x

Adding and subtracting $\left(\frac{a+b}{2}\right)^2$

$= x^2 - x(a+b) + \left(\frac{a+b}{2}\right)^2 - \left(\frac{a+b}{2}\right)^2 + ab$

$= \left(x - \frac{a+b}{2}\right)^2 - \frac{(a+b)^2 - 4ab}{4}$

$= \left(x - \frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$ ...(ii) $\left(\because (a+b)^2 - 4ab = a^2 + b^2 + 2ab - 4ab = a^2 + b^2 - 2ab = (a-b)^2\right)$

Putting this value in (i),

$I = \int \frac{1}{\sqrt{\left(x - \frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2}}\, dx$

$= \log \left| x - \frac{a+b}{2} + \sqrt{\left(x - \frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2} \right|$

− +− − 22

+ c 2 1 = log| + − |

∵ ∫ 2 2

dx x x a 2 2 x a −

. 37

a b

+

x xx a b ab

= log 2 ( ) 2

− + − ++ + c [By (ii)]

Note. Method to evaluate ∫ Linear

dx or ∫ Linear d

Quadratic

Quadratic dx

or ∫ Linear Quadratic dx.

dx (Quadratic) +

B.

Write linear = A

Find values of A and B by comparing coefficients of x and constant terms on both sides.

x

4 +1

16. 2

x x

2 + –3

+

4 1

x

∫ dx ...(i)

Sol. Let I = 2

+ −

2 3
x x

d

Here $\frac{d}{dx}$(Quadratic $2x^2 + x - 3$) is (4x + 1), the numerator. So

put $2x^2 + x - 3 = t$.

dt

$\therefore (4x + 1) = \frac{dt}{dx} \Rightarrow (4x + 1)\,dx = dt$ dt 1/2

$\therefore$ From = t∫= 1/2

(i), I = dt t^{-} ∫

t+ c 1 2

= 2

t+ c = 2^2 2 3 x x + − + c.

17. $_2$ + 2 − 1

x

x

1

x

x

x

x

−∫

dx

Sol. Let I = $_2{}^2$1

dx =$_{2\,2}$2 +

1 1

=^{2}1 x

dx + $2_{2\,2}$ dx

x − ∫ x − 1 ∫

=^{2}1

dx + 2 log | x + $^{2\,2}$ x − 1 | + c x − ∫

...(i)

x

x – ∫

x

dx = $\frac{1}{2}$ 2^2 1

Let I_1 = 2 1 dx dt

x – ∫ dt

Put $x^2 - 1 = t$. Therefore

2x = $\frac{dt}{dx}$ or 2x dx = dt ∫

t^{-}

2

x

t ∫ = $\frac{1}{2}$ $2^{1/2}$ dt

∴ $I_1 = \frac{1}{2}$

t = t = $\sqrt{x^2 - 1}$ 1

= $\frac{1}{2^{1/2}}$

Putting this value of (I_1 =) 2 1 dx = $\sqrt{x^2 - 1}$ in (i)

I = $\sqrt{x^2 - 1}$ + 2 log | x + $\sqrt{x^2 - 1}$ | + c.

18. $\frac{5 - 2x}{1 + 2x + 3x^2}$

Linear

–

5 2

x – ∫ + + ∫ d x

dx ...(i)

i.e., 5x – 2 = A

dx(1 + 2x + 3x2) + B

Sol. Let I = 2

12 3

x x

A

dx(Quadratic)

+ B

Quadratic ∫

dx

Let Linear = d

or 5x – 2 = A(2 + 6x) + B ...(ii) i.e., 5x – 2 = 2A + 6Ax + B

Comparing coefficients of x, 6A = 5 ⇒ A = 5 6

Comparing constants, 2A + B = – 2

Putting A = 5 6 , 10 6 + B = – 2

– – or B = 11

⇒ B = – 2 – 10 6 = 22

3

6

Putting values of A and B in (ii), 5x – 2 = 5 6 (2 + 6x) – 11 3 Putting this value of 5x – 2 in (i),

5 11 (2 6) 6 3

x

+ –

+ +∫

dx

I = 2

12 3

x x

+

2 6 x 1

⇒ I = 5 6 2

+ +∫ dx – 11 3 2

12 3 + +

12 3 x x

x x ∫ dx

= 5 6 I 1 – 11 3 I2...(iii) +

2 6 x

+ +∫ dx

Here I1 = 2

12 3

x x

Put Denominator $1 + 2x + 3x^2 = t$.

dt

$\therefore 2 + 6x = dt$ = dt

dx $\Rightarrow$ (2 + 6x) dx

$\therefore$ I 1 = 1

t∫ dt = log | t | = log | 1 + 2x + $3x^2$|

...(iv) 12 3 + + x x ∫ dx = $_2$1

t∫ =1

Again I_2 = 2

3 21 x x + +∫ dx 1 Quadratic ∫ dx

Now Quadratic Expression = $3x^2 + 2x + 1$. Making

coefficient of x^2 unity = 3 2 2 1 + +

3 3 x x

2

+ + +−

$_2$2 1 11

Completing squares = 3

3 3 39 x x

1 1 31 2

=

2 1 2 3 3 9 x − ∵ −= = 39 9 9

+ +

dx3 3 x

∫ dx = 1

∫ 3^2 $_2$1

+ +

1

$\Rightarrow I_2$ = 2 1 2

1 2 3 3 9 x

+ +

1

1 3 1 2 3

1 1 tan

x + 1 – 3

x dx

∵ $\int$

+

tan⁻ = 2 2

1 3 x a a a

2

=

2 x +

x + = 1 2 tan – 1 3 1

⇒ I 2 = 1 3 . 3 2 tan – 1 3 1

...(v) 2

Putting values of I_1 and I_2 from (iv) and (v) in (iii), we have I = $\frac{5}{6}$ log |

1 + 2x + 3x² | – 11 3 1 2 tan – 1 3 1

x +

+ c.

2

Integrate the functions in Exercises 19 to 23:

19. 6 +7

x

(– 5)(– 4)

x x

+

– – $\int$ dx = 2 6 7

Sol. Let I =

+

6 7

x

(5)(4) x x

$\int$ dx – – +

x

i.e., I = 2 6 7

$\int$ dx ...(i) Linear

+

x

x xx 4 5 20

x x
9 20

Quadratic $\int$
dx

Let Linear = A $\frac{d}{dx}$(Quadratic) + B

− +

i.e., 6x + 7 = A(2x – 9) + B ...(ii) = 2Ax – 9A + B

Comparing coefficients of x, 2A = 6 ⇒ A = 3

Comparing constants, – 9A + B = 7.

Putting A = 3, – 27 + B = 7 ⇒ B = 34

Putting values of A and B in (ii),

6x + 7 = 3(2x – 9) + 34

Putting this value of 6x + 7 in (i),

− +
3(2 9) 34
x
dx 9 20
−
I =2
$\int$
− +

x x

dx + 34 21
x
9 20
− +
x x − +
x x
9 20
dx

= 3 22 9 $\int$ $\int$

= 3 I_1 + 34 I_2...(iii) −

I_1 = 22 9 dx
x
− +
$\int$

x x
9 20

Put x^2– 9x + 20 = t. ∴ 2x – 9 =

⇒ (2x – 9) dx = dt

dt

dt

$\int = {}_{1/2}$

dx

. 40

$\therefore I^{1} = \quad t\int =^{1/2} \quad t\ dt^{-}$

t = 2
t 1/2

= 2^2 x x − + 9 20 ...(iv)

$I_2 = {}_2 1 1$
x x − + 9 20 $\int$ dx = 2

$\int$
dx

9 81 9 20

 −+ +−

2 4 x x
2

1
1

$\int$ dx
$\int$

dx = 2 2

= 2 9 1   2 2 9 91
 − −   − −   2
       2 4 x

9 1

2 x

= log

  −+ − −   dx x x a log| |

1

2 22 x x

∵ $\int$ 2 2

  = +−

2 2

−

x a

I 2 = log9 2 9 20

2 x xx −+ − + ...(v) 2 2 9 1 81 1 2 2 9 9 20
+
22 4 4 x x x xx ∵

− − = + − −= −

Putting values of I_1 and I_2 from (iv) and (v) in (iii), $I = 6^2$
x x − + 9 20
+ 34 log9 2 9 20

2 x xx −+ − + + c.

x + 2

20. 2
4 –
x x

...(i) Linear

– ∫
dx

dx 2
x
+

Quadratic ∫

Sol. Let I = 2 4 x x

Let Linear = A $\frac{d}{dx}$ (Quadratic) + B

i.e., x + 2 = A(4 – 2x) + B ...(ii) = 4A – 2Ax + B

Comparing coefficients of x: – 2A = 1 ⇒ A = $-\frac{1}{2}$
Comparing constants: 4A + B = 2

Putting A = $-\frac{1}{2}$, – 2 + B = 2 ⇒ B = 4

Putting values of A and B in (ii), x + 2 = $-\frac{1}{2}$(4 – 2x) + 4 Putting this value of x + 2 in (i),

1 (4 2) 4 2

− + −

x

$-\int$

dx = 1 2^{-2} $-$

4 2 x 1 dx + 4_2

$-\int$

I = ^{2} 4 x x4x x $-\int$ 4 x x dx

= $\frac{1}{2}$ $-$ $I_1 + 4 I_2$...(iii) $I_1 = {}_2 -$ dx

4 2 x

dt = $_{1/2}$

Put $4x - x^2 = t \therefore 4 - 2x = \mathrm{dt}$

dx $\Rightarrow$ (4 – 2x) dx = dt$\int$ I_2 =

$-\int$

4 x x

2

dt

$\therefore I^{1} =$ t$\int$ $=^{1/2}$ t^{-} 1

dx ...(iv) 1/2

4x x $-\int$

t= 2 t = 2^2 4x x $-$

Quadratic Expression is $4x - x^2 = -x^2 + 4x$

$= -(x^2 - 4x) = -(x^2 - 4x + 4 - 4) = -((x-2)^2 - 2^2) = 2^2 - (x-2)^2$

1

x $^{-}$ ...(v)

dx = sin– 1_2

$\therefore I_2 = {}_{2\,2}$

1 sin x

2 2 (2) – – dx

x $\int$

$\because \int {}^{1}{}_{2\,2}$ a x a

Putting values of I_1 and I_2 from (iv) and (v) in (iii), x – +

c.

I = –2 4x x − + 4 sin− 12

2

21. 2+ 2

x

x x

+2 +3

x+ 2

Sol. Let I =

∫ dx ...(i)

2

+ +

x x

2 3

Let Linear = A $\frac{d}{dx}$(Quadratic) + B

i.e., x + 2 = A(2x + 2) + B ...(ii) = 2Ax + 2A + B

Comparing coefficients of x, 2A = 1 ⇒ A = $\frac{1}{2}$

Comparing constants, 2A + B = 2

Putting A = $\frac{1}{2}$, 1 + B = 2 ⇒ B = 1

Putting values of A and B in (ii), x + 2 = $\frac{1}{2}$(2x + 2) + 1 Putting this value of (x + 2) in (i),

1 (2 2) 12

x

+ ++

∫

2 3∫

x x

dx + 2 2 3 . 42

I =2

x x

2 3

+ +

+ +

dx

dx = 1 222 2

x x + +

x

∫

+

$\Rightarrow$ I = $\frac{1}{2}$ I_1 + I_2...(iii) I_1 = $_2$2 2

x

dx x x

+ + ∫

(2x + 2) = dt

dt∫ 2

dt

2 3

Put x^2+ 2x + 3 = t ∴ dx ⇒ (2x + 2) dx = dt = $_{1/2}$

t∫ =$^{1/2}$

I^1 = t$^-$

t= 2 t = 2^2 x x + + 2 3 ...(iv) 1

x x + + dx =

2

$_2$1

I$_2$ = $_2$1 1

3 ∫

2 12 ∫

x x + ++ dx

dx = log | x + 1 + $^{2\ 2}$ (1) (2) x +

+ |

= $_{2\ 2}$

∫

(1) (2) x + +

1

dx x x a log| |

∵ ∫$^{2\ 2}$

$_{2\ 2}$ = ++ $^+$

x a

= log | x + 1 +^{2} x x + + 2 3 | ...(v) Putting values from (iv) and (v) in (iii),

I =^{2} x x + + 2 3 + log | x + 1 +^{2} x x + + 2 3| + c.

22. $_2$+ 3

x

–2 –5

x x

+

Sol. Let I

= $_2$3 x

– – ∫ d dx ...(i) 2x x

5

dx $(x^2$ – 2x – 5) +

B

Let x + 3 = A

or x + 3 = A(2x – 2) + B ...(ii) = 2Ax – 2A + B Comparing

coefficients of x on both sides, 2A = 1 ⇒ A = $\frac{1}{2}$ Comparing constants, – 2A + B = 3

Putting A = $\frac{1}{2}$, – 1 + B = 3 ⇒ B = 4

Putting values of A and B in (ii), x + 3 = this value in (i),

x

– + –

1 (2 2) 4 2

$\frac{1}{2}$(2x – 2) + 4 Putting

dx = $\frac{1}{2}$ 2 2 – – ∫

I = 2 x x

2 5 x x

2

2 5

x x – –

2 5 ∫

dx

– – ∫

dx + 4

x

2 1

= $\frac{1}{2}I_1 + 4 I_2$...(iii) –

I_1 = 2 2 2 x

– – ∫ dx

x x

2 5

dx ⇒ (2x – 2) dx = dt . 43 Class 12

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Put x^2– 2x – 5 = t. Therefore (2x – 2) = dt

∴ I 1 =

t ∫ = log | t | = log | x^2 – 2x – 5 |

...(iv) Again I_2 = 2 1

dt

x x – – 2 dx

5 ∫

= 2 1 1

x x – + – –

2 115 ∫ dx = 2 (1) 6 x – – ∫

1

dx

(1) 6 x – – ∫ =
1

2 6 log1 6
1 6

= 2

– +...(v) x
dx
x
– –

∵ ∫

dx x+a
x a a

x–a

1 1 = log– 2

+ 4 + 10
x

2 2

Putting values of I_1 and I_2 from (iv)

and (v) in (iii), x – –

I = $\frac{1}{2}$log | x^2 – 2x – 5 | + ^{2}6 log1 6
+

– + + c. 1 6

23. 25 +3
x

x

x x

Sol. Let I = 25 3

∫
dx ...(i)+

+ x x
4 10

d

Let Linear = A B
dx(Quadratic) +

i.e., 5x + 3 = A(2x + 4) + B ...(ii) = 2Ax + 4A + B

Comparing coefficients of x on both sides, 2A = 5 ⇒ A = 5 2
Comparing constants, 4A + B = 3

Putting A = 5 2, 10 + B = 3 ⇒ B = – 7

Putting values of A and B in (ii), 5x + 3 = 5 2(2x + 4) – 75 (2 4)

7 2

x + −

dx

∫

Putting this value in (i), I

= 2

x x

+ +

dx – 7

∫

21

=

x

5

222 4

4 10

dx

x x + +

+

4 10 ∫ + +

x x

4 10

52I 1 – 7 I2...(iii) +

2 4

x

∫ dx

I1 = 2

+ +

x x

4 10

or I =

+ 4 =

dt

dx ⇒ (2x + 4) dx = dt . 44

Put x^2+ 4x + 10 = t. Therefore 2x

dt = 1/2

dt

$t\int = ^{1/2}$

t^{-}

$\therefore I^{1} =$

t = 2 t 1

$= 2^2$ x x + + 4 10 ...(iv) $I_2 = {}_2 1$ 1

x x + + 4 10 $\int$ dx = ${}_2 1$

dx

x x + ++ 4 46 $\int$

$\int$

dx = log | x + 2 + 2 2 (2) (6) x + (2) (6) x + +

+ |

= 2 2

1

dx x x a log| |

$\because \int^{2\,2}$

2 2 = ++ +

x a

= log | x + 2 + ^{2} x x + + 4 10 | ...(v) Putting values of I_1 and I_2 from (iv) and (v) in (iii),

I = 5^2 x x + + 4 10 – 7 log | x + 2 + ^{2} x x + + 4 10 | + c. Choose the correct answer in Exercises 24 and 25. dx

24. $\int^2$ +2 +2

x x equals

(A) x $\tan^{-1}$(x + 1) + C (B) $\tan^{-1}$(x + 1) + C (C) (x + 1) $\tan^{-1}$ x + C

(D) $\tan^{-1}$ x + C.

x x + + $\int$ = ${}_2 1$

1

dx

dx = 2 2

correct answer. dx x x equals

Sol. 2 2 2

x x + ++ 2 11 $\int$

25. $\int^2$ 9 –4

(1) 1 x + + $\int$ –

dx
9 –8
1 1 tan
x dx
x a a a
x + + C1
= 1 1tan − 1(1)
1
= tan − 1 (x + 1) + C ∴ 8 –9

Option (B) is the

(A) 1 9sin − 1 8
9 –8
x + C (B) 1 2sin − 1
9 –8
(C) 1 3sin − 1 8
x + C (D) 1 2sin − 1
x + C.
8
dx
dx
x x − ∫ = 2 4 9
x + C 9

Sol. Let I = 2 9 4 − + x x ∫ ...(i) Here Quadratic expression is $-4x^2$ +
9x = − 4 2 9 4 x x −
−+ −
= − 4 22 9 9
= − 4
2 2
8 8 x
99 9
2 48 8 x x
− −
8 8 x −− + = 4 2 2 9 9 = 4
2 2 9 9 8 8 x . 45
− −

Class 12 Chapter 7 - Integrals Putting this value in (i),

1
1
I = 2 2 − −
∫ dx = 1 2 2 2

9 9 4 8 8

9

8

x

9 8

$= \frac{1}{2}\sin^{-1} x -$ + C1 sin

$\int dx$ 8 8 x

x dx a

a

$\because \int$

x

x C1

$= \frac{1}{2}\sin^{-1}$ 8 9 9

∴ Option (B) is the correct answer.

= 2 2

Exercise 7.5

Integrate the (rational) functions in Exercises 1 to 6: x

1. (+1)(+ 2)

x x

x

Sol. To integrate the (rational) function (1)(2) x x + +.

x x + + = A

x

Let integrand (1)(2) x + 1 + B x + 2 ...(i)

(Partial Fractions)

Multiplying by L.C.M. = (x + 1)(x + 2),

x = A(x + 2) + B(x + 1) = Ax + 2A + Bx + B

Comparing coefficients of x on both sides, A + B = 1 ...(ii) Comparing constants, 2A + B = 0 ...(iii) Let us solve Eqns. (ii) and (iii) for A and B.

Eqn. (iii) – Eqn. (ii) gives, A = – 1

Putting A = – 1 in (ii), – 1 + B = 1 ⇒ B = 2

x

+ + 2

x x + + = 1

Putting values of A and B x 1

x + 2

in (i), (1)(2) x

x + 1 ∫ dx + 2 1

x x + + ∫ dx = – 1 –

∴ (1)(2)

2. $_2$1

x – 9

= – log | x + 1 | + 2 log | x (∵ | t $|^2$ = t^2)

+ 2 | + c

x + 2 ∫ dx^2(2)

x

= log | x + 2 $|^2$ – log | x + 1

| + c = log

\+

++ c.

x − 9 ∫ dx = 2 2

Sol. To integrate the (rational) function 2 1

x − 9

| 1| x

1

1

x − 3 ∫ dx

2

∵ ∫

x

x

= 1

1 1 = log− 2 x−a

+ + c 2 2

−

2 3 . log 3 3 x

x a a x+a

1 3

= 6 log 3 −

x

+ + c.

OR

x − 9 = 1 x − 3 + B

Integrand 2 1

(3)(3) x x − +

=

x + 3

A

Now proceed as in the solution of Q.No.1. 3.3

–1

x

xxx.

(– 1)(– 2)(– 3)

x

Sol. To integrate the (rational) function 3 1

–

xxx

(1)(2)(3)

–––

––– = A

x – 1 + B

x – 3...(i)

Let integrand 3 1 –

xxx

(1)(2)(3) x – 2 + C x

Multiplying by L.C.M. = (x – 1)(x – 2)(x – 3), we have 3x – 1 = A(x – 2)(x – 3) + B(x – 1)(x – 3) + C(x – 1)(x – 2) = A(x^2 – 5x + 6) + B(x^2 – 4x + 3) + C(x^2 – 3x + 2) = Ax^2 – 5Ax + 6A + Bx^2 – 4Bx + 3B + Cx^2 – 3Cx + 2C Comparing coefficients of x^2, x and constant terms on both sides, we have

Coefficients of x^2: A + B + C = 0 ...(ii) Coefficient of x: – 5A – 4B – 3C = 3 or 5A + 4B + 3C = – 3 ...(iii) Constants: 6A + 3B + 2C = – 1 ...(iv) Let us solve (ii), (iii) and (iv) for A, B, C.

Let us first form two Eqns. in two unknowns say A and B. Eqn. (iii) – 3 Eqn. (i) gives (to eliminate C),

5A + 4B + 3C – 3A – 3B – 3C = – 3

or 2A + B = – 3 ...(v) Eqn. (iv) – 2 Eqn. (i) gives (to eliminate C),

6A + 3B + 2C – 2A – 2B – 2C = – 1

or 4A + B = – 1 ...(vi) Eqn. (vi) – Eqn. (v) gives (to eliminate B),

2A = – 1 + 3 = 2 ⇒ A = $\frac{2}{2}$ = 1.

Putting A = 1 in (v), 2 + B = – 3 ⇒ B = – 5

Putting A = 1 and B = – 5 in (ii), 1 – 5 + C = 0

or C – 4 = 0 or C = 4

Putting values of A, B, C in (i),

3 1

= 1

x

−

x − 3

x −

2 + 4

(1)(2)(3)

−

xxx

x

x −

1 _ 5

. 48

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∴3 1

--- ∫

xxx

(1)(2)(3)

=1

x − 1 ∫ dx

dx − 5 x − 2 ∫

1

1

dx + 4 x − 3 ∫

= log | x − 1 | − 5 log | x − 2 | + 4 log | x − 3 | + c. x

4. (− 1)(− 2)(− 3)

xxx

x

Sol. To integrate the (rational) function (1)(2)(3) xxx ---.

2)(

x

x − 2 + C

=

xxx ---

A

Let integrand (1)(+

x − 3 ...(i)

3) x − 1

B

(Partial fractions)

Multiplying by L.C.M. = (x − 1)(x − 2)(x − 3),

x = A(x − 2)(x − 3) + B(x − 1)(x − 3) + C(x − 1)(x − 2) = A(x²

$-5x+6)+B(x^2-4x+3)+C(x^2-3x+2)$

$= Ax^2-5Ax+6A+Bx^2-4Bx+3B+Cx^2-3Cx+2C$ Comparing coefficients of x^2, x and constant terms on both sides, we have x^2: A + B + C = 0 ...(ii) x: – 5A – 4B – 3C = 1 or 5A + 4B + 3C = – 1 ...(iii) Constants: 6A + 3B + 2C = 0 ...(iv) Let us solve Eqns. (ii), (iii) and (iv) for A, B, C.

Let us first form two Eqns. in two unknowns say A and B. Eqn. (iii) – 3 × Eqn. (ii) gives | To eliminate C 5A + 4B + 3C – 3A – 3B – 3C = – 1 or 2A + B = – 1 ...(v) Eqn. (iv) – 2 × Eqn. (ii) gives | To eliminate C 4A + B = 0 ...(vi) Eqn. (vi) – Eqn. (v) gives (To eliminate B)

$2A = 1 \therefore A = \frac{1}{2}$

Putting $A = \frac{1}{2}$ in (v), $1 + B = -1 \Rightarrow B = -2$

Putting $A = \frac{1}{2}$ and B = – 2 in (ii),

$\frac{1}{2} - 2 + C = 0 \Rightarrow C = -\frac{1}{2} + 2 = \frac{-1+4}{2} = \frac{3}{2}$

Putting these values of A, B, C in (i), we have

$\frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} - \frac{2}{x-2} + \frac{3}{2(x-3)}$

$\therefore \int \frac{x}{(x-1)(x-2)(x-3)}\,dx$

$= \frac{1}{2}\int \frac{1}{x-1}\,dx - 2\int \frac{1}{x-2}\,dx + \frac{3}{2}\int \frac{1}{x-3}\,dx$

$= \frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + c.$. 49

5. $\frac{2x}{x^2+3x+2}$

Sol. To integrate the (rational) function $_2{}^2 3\ 2$

x

x x + +.

Now $x^2 + 3x + 2 = x^2 + 2x + x + 2 = x(x + 2) + 1(x + 2) = (x + 1)(x + 2)$

x

∴ Integrand $_2{}^2 3\ 2$

x

x x + + = 2

x x + +

(1)(2)

= A

x +

1 + B

x + 2...(i)

(Partial Fractions)

Multiplying both sides by L.C.M. = (x + 1)(x + 2), 2x = A(x + 2) + B(x + 1) = Ax + 2A + Bx + B Comparing coefficients of x and constant terms on both sides, we have

Coefficients of x: A + B = 2 ...(ii) Constant terms: 2A + B = 0 ...(iii) Let us solve (ii) and (iii) for A and B.

(iii) – (ii) gives A = – 2.

Putting A = – 2 in (ii), – 2 + B = 2. ∴ B = 4 x x + + =2

+ + 4

x –

Putting values of A and

B in (i), $_2{}^2 3\ 2$ x x + +∫ dx = dx + 4 1

x

∴ $_2{}^2 3\ 2$ – 21 x + 1 ∫ x 1 x + 2 x + 2 ∫

dx

= – 2 log | x + 1 | + 4 log | x + 2 | + c = 4 log | x + 2 | – 2 log | x + 1 | + c

Remark: Alternative method to evaluate $_2{}^2 3\ 2$

x

dx

x x + + ∫

is Linear

Quadratic ∫ (1 – 2)

dx as explained in solutions in Exercise 7.4

(Exercise 18 and Exercise 22. 2 1 –

x 2 1 2 2

6.

x x

– x 1 x x 1

Sol. To integrate (rational)

– = 2

function = x x – + 2

–

(1 2) x x 2 x x

2 – +

[Here Degree of numerator = Degree of Denominator = 2 ∴ We must divide numerator by denominator to make the degree of numerator smaller than degree of denominator so that we can form partial fractions.]

$-2x^2 + x$) $-x^2 + 1$ ($\frac{1}{2}$

$-x^2 + \frac{x}{2}$

+ –

$-\frac{x}{2} + 1$

x

2 1

Divisor = $\frac{1}{2}$ + – 1

– x

+

– = Quotient + Remainder ∴

x x

(1 2)

x

− +

2

−

x x (1 2)